
Evacuation Dynamics: Empirical Results, Modeling and Applications

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1 Definition of the subject and its importance

Today, there are many occasions where a large number of people gathers in a rather small area. Office buildings and apartment house become larger and more complex. Very large events related to sports, entertainment or cultural and religious events are held all over the world on a regular basis. This brings about serious safety issues for the participants and the organizers who have to be prepared for any case of emergency or critical situation. Usually in such cases the participants have to be guided away from the dangerous area as fast as possible. Therefore the understanding of the dynamics of large groups of people is very important.

In general, evacuation is the egress from an area, a building or vessel due to a potential or actual threat. In the cases described above the dynamics of the evacuation processes is quite complex due to the large number of people and their interaction, external factors like fire etc., complex building geometries,... Evacuation dynamics has to be described and understood on different levels: physical, physiological, psychological, and social. Accordingly, the scientific investigation of evacuation dynamics involves many research areas and disciplines. The system “evacuation process” (i.e. the population and the environment) can be modelled on many different levels of detail, ranging from

hydro-dynamic models to artificial intelligence and multi-agent systems. There are at least three aspects of evacuation dynamics that motivate its scientific investigation: 1) as in most many-particle systems several interesting collective phenomena can be observed that need to be explained; 2) models need to be developed that are able to reproduce pedestrian dynamics in a realistic way, and 3) the application of pedestrian dynamics to facility design and emergency preparation and management.

The investigation of evacuation dynamics is a difficult problem that requires close collaboration between different fields. The origin of the apparent complexity lies in the fact that one is concerned with a many-‘particle’ system with complex interactions that are not fully understood. Typically the systems are far from equilibrium and so are e.g. rather sensitive to boundary conditions. Motion and behaviour are influenced by several external factors and often crowds can be rather inhomogeneous.

In this article we want to deal with these problems from different perspectives and will not only review the theoretical background, but also discuss some concrete applications.

2 Introduction

The awareness that emergency exits are one of the most important factors to ensure the safety of persons in buildings can be traced more than 100 years. The disasters due to the fires in the Ringtheater in Vienna and the urban theater in Nizza at 1881 with several hundred fatalities lead to a rethinking of the safety in buildings [25]. Firstly it was tried to improve safety by using non-flammable building materials. However, the disaster at the Troquois Theater in Chicago with more than 500 fatalities, where only the decoration burned, caused a rethinking. It was a starting point for studying the influences of emergency exits and thus the dynamics of pedestrian streams [25, 33].

In recent years there were mainly two incidents including evacuations which gained immense global attention. First there was the capsizing of the Baltic Sea ferry MV Estonia (September 28, 1994, 852 casualties) [100] and then of course the terrorist attacks of 9/11 (2,749 casualties). Other prominent examples of the possible tragic outcomes of the dynamics of pedestrian crowds are the Hillsborough stadium disaster in Sheffield (April 15, 1989, 96 casualties) [183], the accident at Bergisel (December 4, 1999, 5 casualties) [190], the stampede in Baghdad (August 30, 2005, 1,011 casualties), the tragedy at the concert of “The Who” (December 3, 1979, 11 casualties) [73] and – very early – the events at the crowning ceremony of Tsar Nicholas II. in St. Petersburg in May 1896 with 1,300 to 3,000 fatalities (sources vary considerably) [169]. In the past tragic accidents have happened frequently in Mecca during the Hajj (1990: 1,426, 1994: 270, 1997: 343, 1998: 107, 2001: 35, 2003: 14, 2004: 244, and 2006: 364 casualties). What stands out is that the initiating events are

very diverse and span from external human aggression (terrorism) over external physical dangers (fire) and rumors to various shades of greedy behavior in absence of any external danger.

Many authors have pointed out that the results of experts' investigations and the way the media typically reports about an accident very often differ strongly [17, 77, 109, 155, 156, 179]. The public discussion has a much greater tendency to identify "panic" as cause of a disaster, while expert commissions often conclude that there either was no panic at all, or panic was merely a result of some other preceding phenomenon.

The article first discusses the empirical basis of pedestrian dynamics in Sec. 3. Here we introduce the basic observables and describe the main qualitative and quantitative results, focussing on collective phenomena and the fundamental diagram. It is emphasized that even for the most basic quantities no consensus about the basic behaviour has been reached.

In Sec. 4 various model approaches that have been applied to the description of pedestrian dynamics are reviewed.

Sec. 5 discusses more practical issues and gives a few examples for applications to safety analysis. In this regard, prediction of evacuation times is an important problem as often legal regulations have to be fulfilled. Here commercial software tools are available. A comparison shows that the results have to be interpreted with care.

3 Empirical Results

3.1 Overview

Pedestrians are three-dimensional objects and a complete description of their highly developed and complicated motion sequence is rather difficult. Therefore usually in pedestrian and evacuation dynamics the motion is treated as two-dimensional by considering the vertical projection of the body.

In the following sections we review the present knowledge of empirical results. These are relevant not only as basis for the development of models, but also for applications like safety studies and legal regulations.

We start with the phenomenological description of collective effects. Some of these are known from everyday experience and will serve as benchmark tests for any kind of modelling approach. Any model that does not reproduce these effects is missing some essential part of the dynamics. Next the foundations of a quantitative description are laid by introducing the fundamental observables of pedestrian dynamics. Difficulties arise from different conventions and definitions. Then pedestrian dynamics in several simple scenarios (corridor, stairs etc.) is discussed. Surprisingly even for these simple cases no consensus about the basic quantitative properties exists. Finally, more complex scenarios are discussed which are combinations of the simpler elements. Investigations of scenarios like evacuations of large buildings or ships suffer even more from lack of reliable quantitative and sometimes even qualitative results.

3.2 Collective Effects

One of the reasons why the investigation of pedestrian dynamics is also attractive for physicists is the large variety of interesting collective effects and self-organization phenomena that can be observed. These macroscopic effects reflect the individuals' microscopic interactions and thus give also important information for any modelling approach.

- **Jamming**

Jamming and clogging typically occur for high densities at locations where the inflow exceeds the capacity. Locations with reduced capacity are called *bottlenecks*. Typical examples are exits (Fig. 1) or narrowings. This kind of jamming phenomenon does not depend strongly on the microscopic dynamics of the particles. Rather it is a consequence of an exclusion principle: space occupied by one particle is not available for others.

This clogging effect is typical for a bottleneck situation. It is important for practical applications, especially evacuation simulations.



Fig. 1. Clogging near a bottleneck. The shape of the clog is discussed in more detail in Sec. 4.5.

Other types of jamming occur in the case of counterflow where two groups of pedestrians mutually block each other. This happens typically at high densities and when it is not possible to turn around and move back, e.g. when the flow of people is large.

- **Density waves**

Density waves in pedestrian crowds can be generally characterised as quasi-periodic density variations in space and time. A typical example is the

movement in a densely crowded corridor (e.g. in subway-stations close to the density that causes a complete halt of the motion) where phenomena similar to stop-and-go vehicular traffic can be observed, e.g. density fluctuations in longitudinal direction that move backwards (opposite to the movement direction of the crowd) through the corridor. More specifically, for the situation on the Jamarat Bridge in Makkah (during the Hajj pilgrimage 2006) stop-and-go waves have been reported. At densities of 7 persons per m^2 upstream moving stop-and-go waves of period 45 s have been observed that lasted for 20 minutes [60]. Fruin reports, that “at occupancies of about 7 persons per square meter the crowd becomes almost a fluid mass. Shock waves can be propagated through the mass sufficient to lift people of their feet and propel them distances of 3 m (10 ft) or more.” [37].

- **Lane formation**

In counterflow, i.e. two groups of people moving in opposite directions, (dynamically varying) lanes are formed where people move in just one direction [135,139,198]. In this way, strong interactions with oncoming pedestrians are reduced which is more comfortable and allows higher walking speeds.

The occurrence of lane formation does not require a preference of moving on one side. It also occurs in situations without left- or right-preference. However, cultural differences for the preferred side have been observed. Although this preference is not essential for the phenomenon itself, it has an influence on the kind of lanes formed and their order.

Several quantities for the quantitative characterization of lane formation have been proposed. Yamori [198] has introduced a band index which is basically the ratio of pedestrians in lanes to their total number. In [14] a characterization of lane formation through the (transversal) velocity profiles at fixed positions has been proposed. Lane formation has also been predicted to occur in colloidal mixtures driven by an external field [15,29,158]. Here an order parameter $\phi = \frac{1}{N} \left\langle \sum_{j=1}^N \phi_j \right\rangle$ has been introduced where $\phi_j = 1$ if the lateral distance to all other particles of the other type is larger than a typical density-dependent length scale and $\phi_j = 0$ otherwise.

The number of lanes can vary considerably with the total width of the flow. Fig. 2 shows a street in the city center of Cologne during the World Youth Day in Cologne (August 2005) where two comparatively large lanes have been formed.

The number of lanes usually is not constant and might change in time, even if there are relatively small changes in density. The number of lanes in opposite directions is not always identical. This can be interpreted as a sort of spontaneous symmetry breaking.

Quantitative empirical studies of lane formation are rare. Experimental result have been reported in [93] where two groups with varying relative



Fig. 2. The “Hohe Straße” in Cologne during World Youth Day 2005. The yellow line is the border of the two walking directions.

sizes had to pass each other in a corridor with a width of 2 m. On one hand, similar to [198] a variety of different lane patterns were observed, ranging from 2 to 4 lanes. On the other hand, in spite of this complexity surprisingly large flows could be measured: the sum of (specific) flow and counterflow was between 1.8 and 2.8 persons per meter and second and exceeded the specific flow for one-directional motion (≈ 1.4 P/ms).

- **Oscillations**

In counterflow at bottlenecks, e.g. doors, one can sometimes observe oscillatory changes of the direction of motion. Once a pedestrian is able to pass the bottleneck it becomes easier for others to follow in the same direction until somebody is able to pass (e.g. through a fluctuation) the bottleneck in the opposite direction.

- **Patterns at intersections**

At intersections various collective patterns of motion can be formed. A typical example are short-lived roundabouts which make the motion more efficient. Even if these are connected with small detours the formation of these patterns can be favourable since they allow for a “smoother” motion.

- **Emergency situations, “panic”**

In emergency situations various collective phenomena have been reported that have sometimes misleadingly been attributed to *panic behaviour*. However, there is strong evidence that this is not the case. Although a precise accepted definition of *panic* is missing, usually certain aspects are associated with this concept [77]. Typically “panic” is assumed to occur in situations where people compete for scarce or dwindling resources (e.g.

safe space or access to an exit) which leads to selfish, asocial or even completely irrational behaviour and contagion that affects large groups. A closer investigation of many crowd disasters has revealed that most of the above characteristics have played almost no role and most of the time have not been observed at all (see e.g. [73]). Often the reason for these accidents is much simpler, e.g. in several cases the capacity of the facilities was too small for the actual pedestrian traffic, e.g. Lushniki Stadium Moskau (October 20, 1982), Bergisel (December 4, 1999), pedestrian bridge Kobe (Akashi) (July 21, 2001) [187]. Therefore the term “panic” should be avoided, *crowd disaster* being a more appropriate characterisation. Also it should be kept in mind that in dangerous situations it is *not* irrational to fight for resources (or your own life), if everybody else does this [19, 113]. Only from the outside this behavior is perceived as irrational since it might lead to a catastrophe [179]. The latter aspect is therefore better described as *non-adaptive behaviour*.

We will discuss these issues in more detail in Sec. 3.7.

3.3 Observables

Before we review experimental studies in this section, the commonly used observables are introduced.

The flow J of a pedestrian stream gives the number of pedestrians crossing a fixed location of a facility per unit of time. Usually it is taken as a scalar quantity since only the flow normal to some cross-section is considered. There are various methods to measure the flow. The most natural approach is to determine the times t_i at which pedestrians passed a fixed measurement location. The time gaps $\Delta t_i = t_{i+1} - t_i$ between two consecutive pedestrians i and $i + 1$ are directly related to the flow

$$J = \frac{1}{\langle \Delta t \rangle} \quad \text{with} \quad \langle \Delta t \rangle = \frac{1}{N} \sum_{i=1}^N (t_{i+1} - t_i) = \frac{t_{N+1} - t_1}{N}. \quad (1)$$

Another possibility to measure the flow of a pedestrian stream is borrowed from fluid dynamics. The flow through a facility of width b determined by the average density ρ and the average speed v of a pedestrian stream as

$$J = \rho v b = J_s b. \quad (2)$$

where the *specific flow*⁷

$$J_s = \rho v \quad (3)$$

gives the flow per unit-width. This relation is also known as *hydrodynamic relation*.

⁷ In strictly one-dimensional motion often a line density (dimension: 1/length) is used. Then the *flow* is given by $J = \rho v$.

There are several problems concerning the way how velocities, densities or time gaps are measured and the conformance of the two definitions of the flow. The flow according to eq. (1) is usually measured as a mean value over time at a certain location while the measurement of the density in eq. (2) is connected with an instantaneous mean value over space. This can lead to a bias caused by the underestimation of fast moving pedestrians at the average over space compared to the mean value of the flow over time at a single measurement line, see the discussion for vehicular traffic e.g. in [52, 81, 102]. Furthermore most experimental studies measuring the flow according to equation (2) combine for technical reasons a *average* velocity of a single pedestrian over time with an *instantaneous* density. To ensure a correspondence of the mean values the average velocity of all pedestrians contributing to the density at a certain instant has to be considered. However this procedure is very time consuming and not realised in practice up to now. Moreover the fact that the dimension of the test section has usually the same order of magnitude as the extent of the pedestrians can influence the averages over space. These all are possible factors why different measurements can differ in a large way, see discussion in Sec. 3.4.

Another way to quantify the pedestrian load of facilities has been proposed by Fruin [36]. The “pedestrian area module” is given by the reciprocal of the density. Thompson and Marchant [185] introduced the so-called “inter-person distance” d , which is measured between centre coordinates of the assessing and obstructing persons. According to the “pedestrian area module” Thompson and Marchant call $\sqrt{\frac{1}{\rho}}$ the “average inter-person distance” for a pedestrian stream of evenly spaced persons [185]. An alternative definition is introduced in [58] where the local density is obtained by averaging over a circular region of radius R ,

$$\rho(\mathbf{r}, t) = \sum_j f(\mathbf{r}_j(t) - \mathbf{r}), \quad (4)$$

where $\mathbf{r}_j(t)$ are the positions of the pedestrians j in the surrounding of \mathbf{r} and $f(\dots)$ is a Gaussian, distance-dependent weight function.

In contrast to the density definitions above, Predtechenskii and Milinskii [152] consider the ratio of the sum of the projection area f_j of the bodies and the total area of the pedestrian stream A , defining the (dimensionless) density $\tilde{\rho}$ as

$$\tilde{\rho} = \frac{\sum_j f_j}{A}, \quad (5)$$

a quantity known as *occupancy* in the context of vehicular traffic. Since the projection area f_j depends strongly on the type of person (e.g. it is much smaller for a child than an adult), the densities for different pedestrian streams consisting of the same number of persons and the same stream area can be quite different.

Beside technical problems due to camera distortions and camera perspective there are several conceptual problems, like the association of averaged

with instantaneous quantities, the necessity to choose an observation area in the same order of magnitude as the extent of a pedestrian together with the definition of the density of objects with non zero extent and much more. A detailed analysis how the way of measurement influences the relations is necessary but still lacking.

3.4 Fundamental Diagram

The fundamental diagram describes the empirical relation between density ρ and flow J . The name already indicates its importance and naturally it has been the subject of many investigations. Due to the hydrodynamic relation (3) there are three equivalent forms: $J_s(\rho)$, $v(\rho)$ and $v(J_s)$. In applications the relation is a basic input for engineering methods developed for the design and dimensioning of pedestrian facilities [36, 136, 150]. Furthermore it is a quantitative benchmark for models of pedestrian dynamics [21, 83, 112, 177].

In this section we will concentrate on planar facilities like sidewalks, corridors or halls. For various facilities like floors, stairs or ramps the shape of the diagrams differ, but in general it is assumed that the fundamental diagrams for the same type of facilities but different widths merge into one diagram for the specific flow J_s . In first order this is confirmed by measurements on different widths [50, 135, 139, 142]. However, Navin and Wheeler observed in narrow sidewalks more orderly movement leading to slightly higher specific flows than for wider sidewalks [135]. A natural lower bound for the independence of the specific flow from the width is given by the body size and the asymmetry in movement possibilities of the human body. Surprisingly Kretz et al. found an increase of the specific flow for bottlenecks with $b \leq 0.7$ m [92]. This will be discussed in more detail later. For the following discussion we assume facility widths larger than $b = 0.6$ m and use the most common representations $J_s(\rho)$ and $v(\rho)$.

Fig. 3 shows various fundamental diagrams used in planing guidelines and measurements of two selected empirical studies representing the overall range of the data. The comparison reveals that specifications and measurements disagree considerably. In particular the maximum of the function giving the capacity $J_{s,\max}$ ranges from $1.2 (\text{ms})^{-1}$ to $1.8 (\text{ms})^{-1}$, the density value where the maximum flow is reached ρ_c ranges from 1.75 m^{-2} to 7 m^{-2} and, most notably, the density ρ_0 where the velocity approaches zero due to overcrowding ranges from 3.8 m^{-2} to 10 m^{-2} .

Several explanations for these deviations have been suggested, including cultural and population differences [58, 116], differences between uni- and multidirectional flow [99, 135, 154], short-ranged fluctuations [154], influence of psychological factors given by the incentive of the movement [150] and, partially related to the latter, the type of traffic (commuters, shoppers) [139].

It seems that the most elaborate fundamental diagram is given by Weidmann who collected 25 data sets. An examination of the data which were

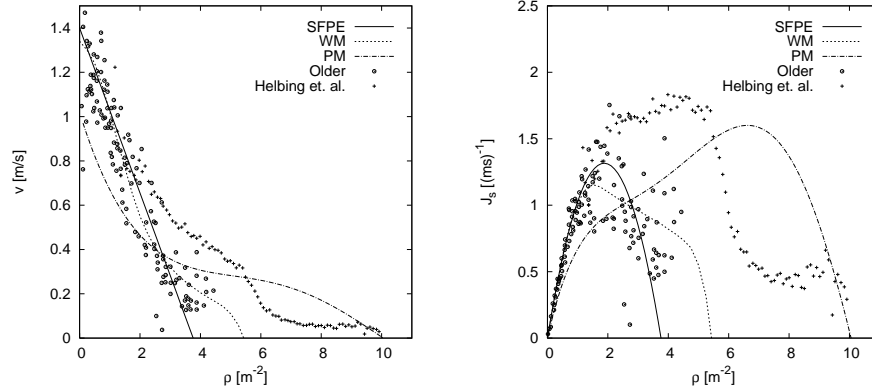


Fig. 3. Fundamental diagrams for pedestrian movement in planar facilities. The lines refer to specifications according to planning guidelines (SFPE Handbook [136]), Predtechenskii and Milinskii (PM) [150], Weidmann (WM) [193]. Data points give the range of experimental measurements (Older [142] and Helbing [58]).

included in Weidmann's analysis shows that most measurements with densities larger than $\rho = 1.8 \text{ m}^{-2}$ are performed on multidirectional streams [135, 139, 140, 142, 148]. But also data gained by measurements on strictly unidirectional streams has been considered [36, 50, 189]. Thus Weidmann neglected differences between uni- and multidirectional flow in accordance with Fruin, who states in his often cited book [36] that the fundamental diagrams of multidirectional and unidirectional flow differ only slightly. This disagrees with results of Navin and Wheeler [135] and Lam et al. [99] who found a reduction of the flow in dependence of directional imbalances. Here lane formation in bidirectional flow has to be considered. Bidirectional pedestrian flow includes unordered streams as well as lane-separated and thus quasi-unidirectional streams in opposite directions. A more detailed discussion and data can be found in [99, 135, 154]. A surprising finding is that the sum of flow and counterflow in corridors is larger than the unidirectional flow and for equally distributed loads it can be twice the unidirectional flow [93].

Another explanation is given by Helbing et al. [58] who argue that cultural and population differences are responsible for the deviations between Weidmann and their data. In contrast to this interpretation the data of Hanking and Wright [50] gained by measurements in the London subway (UK) are in good agreement with the data of Mori and Tsukaguchi [115] measured in the central business district of Osaka (Japan), both on strictly uni-directional streams. This brief discussion clearly shows that up to now there is no consensus about the origin of the discrepancies between different fundamental diagrams and how one can explain the shape of the function.

However, all diagrams agree in one characteristic: velocity decreases with increasing density. As the discussion above indicates there are many possible

reasons and causes for the velocity reduction. For the movement of pedestrians along a line a linear relation between speed and the inverse of the density was measured in [176]. The speed for walking pedestrians depends also linearly on the step size [193] and the inverse of the density can be regarded as the required length of one pedestrian to move. Thus it seems that smaller step sizes caused by a reduction of the available space with increasing density is, at least for a certain density region, one cause for the decrease of speed. However, this is only a starting point for a more elaborated modeling of the fundamental diagram.

3.5 Bottleneck Flow

The flow of pedestrians through bottlenecks shows a rich variety of phenomena, e.g. the formation of lanes at the entrance to the bottleneck [65,66,92,175], clogging and blockages at narrow bottlenecks [25,53,92,121,122,150] or some special features of bidirectional bottleneck flow [53]. Moreover, the estimation of bottleneck capacities by the maxima of fundamental diagrams is an important tool for the design and dimensioning of pedestrian facilities.

Capacity and bottleneck width

One of the most important practical questions is how the capacity of the bottleneck increases with rising width. Studies of this dependence can be traced back to the beginning of the last century [25,33] and are up to now discussed controversially. As already mentioned in the context of the fundamental diagram there are multiple possible influences on pedestrian flow and thus on the capacity. In the following the major findings are outlined, demonstrating the complexity of the system and documenting a controversial discussion over one hundred years.

At first sight, a stepwise increase of capacity with the width appears to be natural if lanes are formed. For independent lanes, where pedestrians in one lane are not influenced by those in others, the capacity increases only if an additional lane can be formed. This is reflected in the stepwise enlargement of exit width which is up to now a requirement of several building codes and design recommendations, see e.g. the discussion in [146] for the USA and GB and [130] for Germany. E.g. the German building code requires an exit width (e.g. for a door) to be 90 cm at least and 60 cm for every 200 persons. Independently from this simple lane model Hoogendoorn and Daamen [65,66] measured by a laboratory experiment the trajectories of pedestrians passing a bottleneck. The trajectories show that inside a bottleneck the formation of lanes occurs, resulting from the zipper effect during entering the bottleneck. Due to the zipper effect, a self-organization phenomenon leading to an optimization of the available space and velocity, the lanes are not independent and thus do not allow passing (Fig. 4). The empirical results of [65,66] indicate a distance between lanes of $d \approx 0.45$ m, independent of the bottleneck

width b , implying a stepwise increase of capacity. However, the investigation was restricted to two values ($b = 1.0$ m and $b = 2.0$ m) of the width.

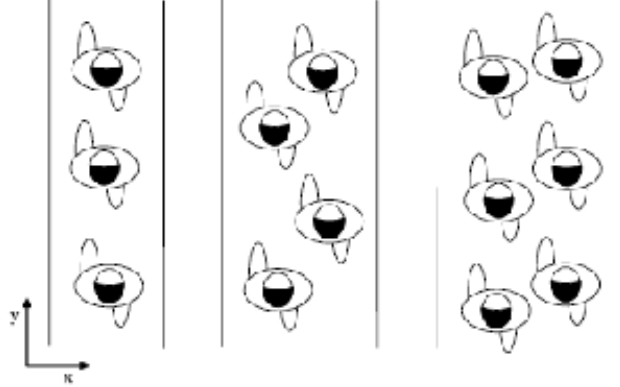


Fig. 4. A sketch of the zipper effect with continuously increasing lane distances in x : The distance in the walking direction decreases with increasing lateral distance. Density and velocities are the same in all cases, but the flow increases continuously with the width of the section.

In contrast, the study [175] considered more values of the width and found that the lane distance increases continuously as illustrated in Fig. 4. Moreover it was shown that the continuous increase of the lane distance leads to a very weak dependence of the density and velocity inside the bottleneck on its width. Thus in reference to eq. (2) the flow does not necessarily depend on the number of lanes. This is consistent with common guidelines and handbooks⁸ which assume that the capacity is a linear function of the width [36, 136, 150, 193]. It is given by the maximum of the fundamental diagram and in reference to the specific flow concept introduced in Section 3.3, eqs. (2), (3), the maximum grows linearly with the facility width. To find a conclusive judgement on the question if the capacity grows continuously with the width the results of different laboratory experiments [92, 121, 122, 131, 175] are compared in [175].

In the following we discuss the data of flow measurement collected in Fig. 5. The corresponding setups are sketched in Fig. 6. First one has to note that all presented data are taken under laboratory conditions where the test persons are advised to move normally. The data by Muir et al. [121], who studied the evacuation of airplanes (see Fig. 6(b)), seem to support the stepwise increase of the flow with the width. They show constant flow values for $b > 0.6$ m. But the independence of the flow over the large range from $b = 0.6$ m to $b = 1.8$ m indicates that in this special setup the flow is not restricted by the bottleneck

⁸ One exception is the German MVStättV [130], see above.

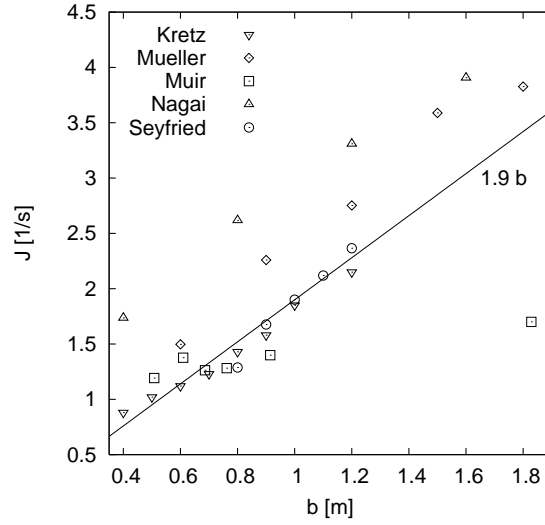


Fig. 5. Influence of the width of a bottleneck on the flow. Experimental data [121, 122, 131, 175] of different types of bottlenecks and initial conditions. All data are taken under laboratory conditions where the test persons are advised to move normally.

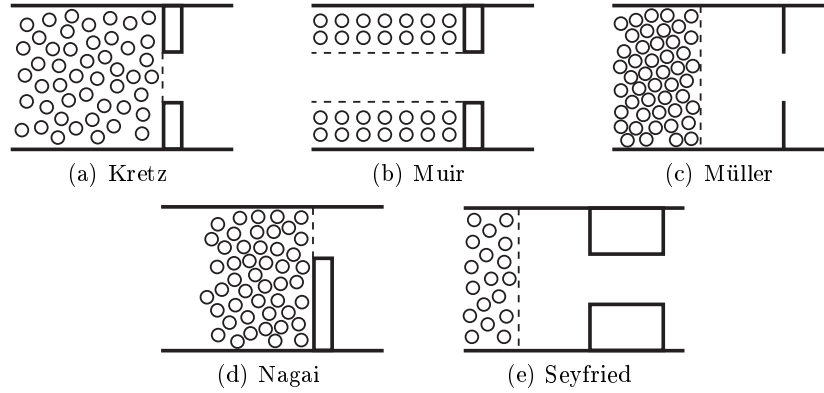


Fig. 6. Outlines of the experimental arrangements under which the data shown in Figure 5 were taken.

width. Moreover it was shown in [175] by determination of the trajectories that the distance between lanes changes continuously, invalidating the basic assumption leading to a stepwise increasing flow. Thus all collected data for flow measurements in Fig. 5 are compatible with a continuous and almost linear increase with the bottleneck width for $b > 0.6$ m.

Surprisingly the data in Fig. 5 differ considerably in the values of the bottleneck capacity. In particular the flow values of Nagai [131] and Müller [122] are much higher than the maxima of empirical fundamental diagrams, see Sec. 3.4. The influence of “panic” or pushing can be excluded since in all experiment the participants were instructed to move normally. The comparison of the different experimental setups (Fig. 6) shows that the exact geometry of the bottleneck is of only minor influence on the flow while a high initial density in front of the bottleneck can increase the resulting flow values. This is confirmed by the study of Nagai et al., see Figure 6 in [131]. There it is shown that for $b = 1.2$ m the flow grows from $J = 1.04$ s⁻¹ to 3.31 s⁻¹ when the initial density is increased from 0.4 m⁻² to 5 m⁻².

The linear dependence of the flow on the width has a natural limitation due to the non-zero body-size and the asymmetry given by the sequence of movement in steps. The moving of pedestrians through bottlenecks smaller than the shoulder width, requires a rotation of the body. Surprisingly Kretz et al. found in their experiment [92] that the specific flow J_s increases if the width decreases from $b = 0.7$ m to $b = 0.4$ m.

Connection between bottleneck flow and fundamental diagrams

An interesting question is how the bottleneck flow is connected to the fundamental diagram. General results for driven diffusive systems [149] show that boundary conditions only *select* between the states of the undisturbed system instead of creating completely different ones. Therefore it is surprising that the measured maximal flow at bottlenecks can exceed the maximum of the empirical fundamental diagram. These questions are related to the common jamming criterion. Generally it is assumed that a jam occurs if the incoming flow exceeds the capacity of the bottleneck. In this case one expects the flow through the bottleneck to continue with the capacity (or lower values).

The data presented in [175] show a more complicated picture. While the density in front of the bottleneck amounts to $\rho \approx 5.0 (\pm 1)$ m⁻², the density inside the bottleneck tunes around $\rho \approx 1.8$ m⁻². The observation that the density inside the bottleneck is lower than in front of the bottleneck is consistent with measurements of Daamen and Hoogendoorn [22] and the description given by Predtechenskii and Milinskii in [150]. The latter assumes that in the case of a jam the flow through the bottleneck is determined by the flow in front of the bottleneck. The density inside the jam will be higher than the density associated with the capacity. Thus the reduced flow in front of the bottleneck causes a flow through the bottleneck smaller than the bottleneck capacity. Correspondingly the associated density is also smaller than that at capacity. But the discussion above can not explain why the capacities measured at bottlenecks are significant higher than the maxima of empirical fundamental diagrams and cast doubts on the common jamming criterion. Possible unconsidered influences are stochastic flow fluctuations, non-stationarity of the flow,

flow interferences due to the necessity of local organization or changes of the incentive during the access into the bottleneck.

Blockages in competitive situations

As stated above all data collected in figure 5 are gained by runs where the test persons were instructed to move normally. By definition a bottleneck is a limited resource and it is possible that under competitive situation pedestrian flow through bottlenecks is different from the flow in normal situations. One qualitative difference to normal situations is the occurrence of blockages. Regarding the term ‘panic’ one has to bear in mind that for the occurrence of blockages some kind of reward is essential while the emotional state of the test persons is not. This was a result of a very interesting and often cited study by Mintz [113]. First experiments with real pedestrians have been performed by Dieckmann [25] in 1911 as reaction to many fatalities in theater fires at the end of the 19th century. In these small scale experiments test persons were instructed to go through great trouble to pass the door as fast as possible. Even in the first run he observed a stable “wedging”. In [150] it is described how these obstruction occurs due to the formation of arches in front of the door under high pressure. This is very similar to the well-known phenomenon of *arching* occurring in the flow of granular materials through narrow openings [195].

Systematic studies including the influence of the shape and width of the bottleneck and the comparison with flow values under normal situations have been performed by Müller and Muir et al. [121,122]. Müller found that funnel-like geometries support the formation of arches and thus blockages. For the further discussion one has to distinguish between temporary blockages and stable blockages leading to a zero flow. For the setup sketched in Fig. 6(c) Müller found that temporary blockages occur only for $b < 1.8$ m. For $b \leq 1.2$ m the flow shows strong pulsing due to unstable blockages. Temporal disruptions of the flow establish for $b \leq 1.0$ m. In comparison to normal situations the flow is higher and in general the occurrence of blockages decrease with width. However a surprising result is that for narrow bottlenecks increasing the width can be counterproductive since it also increases the probability of blockages. Muir et al. for example note that in their setup (Fig. 6(b)) the enlargement of the width from $b = 0.5$ m to $b = 0.6$ m leads to an increase of temporary blockages. The authors explain this by differences in the perception of the situation by the test persons. While the smaller width is clearly passable only for one person the wider width may lead to the perception that the bottleneck is sufficiently wide to allow two persons to pass through. How many people have direct access to the bottleneck is clearly influenced by the width of the corridor in front of the bottleneck. Also Müller found hints that flow under competitive situations did not increase in general with the bottleneck width. He notes an optimal ratio of 0.75 : 1 between the bottleneck width and the width of the corridor in front of the bottleneck.

To reduce the occurrence of blockages and thus evacuation times, Helbing et al. [55, 57, 86] suggested to put a column (asymmetrically) in front of a bottleneck. It should be emphasized that this theoretical prediction was made under the assumption that the system parameters, i.e. the basic behaviour of the pedestrians, does not change in the presence of the column. This is highly questionable in real situations where the columns can be perceived as an additional obstacle or even make it difficult to find the exit. In experiments [53] an increase of the flow of about 30% has been observed for a door with $b = 0.82$ m. But this experiment was only performed for one width and the discussion above indicates the strong influence of the specific setup used. Independent of this uncertainty this concept is limited, as the occurrence of stable arches, to narrow bottlenecks. In practice narrow bottlenecks are not suitable for a large number of people and an opening in a room has also other important functionalities, which would be restricted by a column.

Another surprising finding is the observation that the total flow at bottlenecks with bidirectional movement is higher than it is for unidirectional flows [53].

3.6 Stairs

In most evacuation scenarios stairs are important elements that are a major determinant for the evacuation time. Due to their physical dimension which is often smaller than other parts of a building or due to a reduced walking speed, stairs generally have to be considered as bottlenecks for the flow of evacuees. For the movement on stairs, just as for the movement on flat terrain, the fundamental diagram is of central interest. Compared to the latter one there are more degrees of freedom, which influence the fundamental diagram:

- One has to distinguish between upward and downward movement.
- The influence of riser height and tread length (which determine the incline) has to be taken into account.
- For upward motion exhaustion effects lead to a strong time dependence of the free speed.

It is probably a consequence of the existence of a continuum of fundamental diagrams in dependence of the incline that there are no generally accepted fundamental diagrams for the movement on stairs. However, there are studies on various details — mostly the free speed — of motion on stairs in dependence of the incline [36, 39, 40, 47], conditions (comfortable, normal, dangerous) [152], age and sex [36], tread width [34], and the length of a stair [94]; and in consideration of various disablements [11].

In addition there are some compilations or “meta studies”: Graat [47] compiled a list of capacity measurements and Weidmann [193] built an average of 58 single studies and found an average for the horizontal upstairs speed — the speed when the motion is projected to the horizontal level — of 0.610 m/s.

Depending on the various parameters of aforesaid studies, those studies report horizontal upward walking speeds varying over a wide range from 0.391 to 1.16 m/s. Interestingly on one and the same short stairs it could be observed [94] that people on average walked faster up- than downwards.

There is also a model where the upstairs speed is calculated from the stair geometry (riser and tread) [184] and an empirical investigation of the collision avoidance behavior on stairs [38].

On stairs (up- as well as downward) people like to put their hand on the handrail, i.e. they tend to walk close to walls, even if there is no counterflow. This is in contrast to movement on flat terrain, where at least in situations of low density there is a tendency to keep some distance from walls.

The movement on stairs is typically associated with a reduction of the walking speed. For upward motion this follows from the increased physical effort upward motion requires. This has two aspects, first there is the physical potential energy that a pedestrian has to supply if he wants to rise in height, second the motion process itself is more exertive - the leg has to be lifted higher - than during motion on a level, even if this motion process is executed only on the spot. Concerning the potential energy there is no comparable effect for people going downstairs. But still one can observe jams forming at the upper end of downstairs streams. These are due to the slight hesitation that occurs when pedestrians synchronize their steps with the geometry of the (down-)stairs ahead. Therefore the bottleneck character of downstairs is less a consequence of the speed on the stairs itself and more of the transition from planar to downward movement, at least as long as the steps are not overly steep.

3.7 Evacuations: Empirical results

Up to now this section has focussed on empirical results for pedestrian motion in rather simple scenarios. As we have seen there are many open questions where no consensus has been reached, sometimes even about the qualitative aspects. This becomes even more relevant for full-scale descriptions of evacuations from large buildings or cruise ships. These are typically a combination of many of the simpler elements. Therefore a lack of reliable information is not surprising. In the following we will discuss several complex scenarios in more detail.

Evacuation Experiments

In the case of an emergency, the movement of a crowd usually is more straightforward than in the general case. Commuters in a railway station, for example, or visitors of a building might have complex itineraries which are usually represented by origin-destination matrices. In the case of an evacuation, however, the aims and routes are known and usually the same, i.e. the exits and the egress routes. This is the reason why an evacuation process is rather strictly

limited in space and time, i.e. its beginning and end are well-defined (sound of the alarm, initial position of all persons, safe areas, final position of all persons, and the time, the last person reaches the safe area). When all people have left a building or vessel and reached a safe area (or the lifeboats or liferafts), then the evacuation is finished. Therefore, it is also possible to perform evacuation trials and measure overall evacuation times. Before we go into details, we will clarify three different aspects of data on evacuation processes: (1) the definition and parts of evacuation time, (2) the different sources of data, and (3) the application of these data.

Concerning the evacuation time five different phases can be distinguished [49, 118, 153]: (1) detection time, (2) awareness time, (3) decision time, (4) reaction time, and (5) movement time. In IMO's regulations [118, 119], the first four are grouped together into *response time*. Usually, this time is called *pre-movement time*.

One possible scheme for the classification of data on evacuation processes is shown in the following Fig. 7. Please note that not only data obtained from

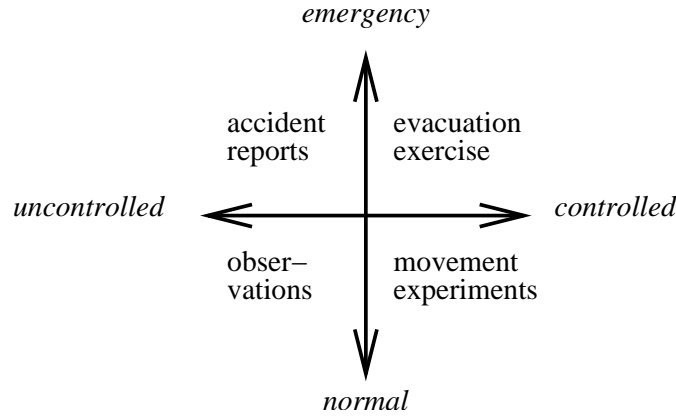


Fig. 7. Empirical data can be roughly classified according to controlled/uncontrolled and emergency/normal situations.

uncontrolled or emergency situations can be used in the context of evacuation assessment. Especially knowledge about bottleneck capacities (i.e. flows through doors and on stairs) is very important when assessing the layout of a building with respect to evacuation. The purpose of empirical data in the context of evacuation processes (and modelling in general) is threefold [44, 71]: (1) identify parameters (factors that influence the evacuation process, e.g. bottleneck widths and capacities), (2) quantify (calibrate) those parameters, e.g. flow through a bottleneck in persons per meter and second, and (3) validate simulation results, e.g. compare the overall evacuation time measured in an evacuation with simulation or calculation results. The validation part

is usually based on data for the evacuation of complete buildings, aircraft, trains or ships. These are available from two different sources: (1) full scale evacuation trials and (2) real evacuations. Evacuation trials are usually observed and videotaped. Reports of real evacuation processes are obtained from eye-witness records and a posteriori incident investigations. Since the setting of a complete evacuation is not experimental, it is hardly possible to measure microscopic features of the crowd motion. Therefore, the calibration of parameters is usually not the main purpose in evacuation trials, but they are carried out to gain knowledge about the overall evacuation process and the behavior of the persons and to identify the governing influences/parameters and validate simulation results.

One major concern in evacuation exercises is the well-being of the participants. Due to practical, financial, and ethical constraints, an evacuation trial cannot be realistic by its very nature. Therefore, an evacuation exercise does not convey the increased stress of a real evacuation. To draw conclusions on the evacuation process, the walking speed observed in an exercise should not be assumed to be higher in a real evacuation [145]. Along the same lines of argument, a simplified evacuation analysis based on e.g. a hydro-dynamic model rather predicts an evacuation exercise and the same constraints apply for its results concerning the prediction of evacuation times and the evacuation process. If the population parameters (like gender, age, walking speed, etc.) are explicitly stated in the model, increased stress can be simulated by adapting these parameters.

In summary, evacuation exercises are just too expensive, time consuming, and dangerous to be a standard measure for evacuation analysis. An evacuation exercise organized by the UK Marine Coastguard Agency on the Ro-Ro ferry “Stena Invicta” held in Dover Harbor in 1996 cost more than 10,000 GBP [117]. This one major argument for the use of evacuation simulations (resp. evacuation analysis based on hydro-dynamic models and calculations).

Panic, Herding, and Similar Conjectured Collective Phenomena

As already mentioned earlier in Sec. 3.2, the concept of “panic” and its relevance for crowd disasters is rather controversial. It is usually used to describe irrational and unsocial behavior. In the context of evacuations empirical evidence shows that this type of behavior is rare [3, 17, 77, 179]. On the other hand there are indications that fear might be “contagious” [23]. Related concepts like “herding” and “stampede” seem to indicate a certain similarity of the behaviour of human crowds with animal behavior. This terminology is quite often used in the public media. *Herding* has been described in animal experiments [166] and is difficult to measure in human crowds. However, it seems to be natural that herding exists in certain situations, e.g. limited visibility due to failing lights or strong smoke when exits are hard to find.

Panic — As stated earlier, “panic” behaviour is usually characterized by selfish and anti-social behaviour which through contagion affects large groups

and even leads to completely irrational actions. Often it is assumed, especially in the media, to occur in situations where people compete for scarce or dwindling resources, which in the case of emergencies are safe space or access to an exit. However, this point of view does not stand close scrutiny and it has turned out that this behaviour has played no role at all in many tragic events [73, 77]. For these incidents *crowd disaster* is then a much more appropriate characterisation.

Furthermore, lack of social behavior seems to be more frequent during so called “acquisitive panics” or “crazes” [180] than during “flight panics”. I.e. social behavior seems to be less stable if there is something to gain than if there is some external danger which threatens all members of a group. Examples for crazes (acquisitive panics) include the Victoria Hall Disaster (1883) [150], the crowning ceremony of Tsar Nicholas II (1896) [169], a governmental Christmas celebration in Aracaju (2001), the distribution of free Saris in Uttar Pradesh (2004), and the opening of an IKEA store in Jeddah (2004). Crowd accidents which occur at rock concerts and religious events as well bear more similarities with crazes than with panics.

However, it is not the case that altruism and cooperation increase with danger. The events during the capsizing of the MV Estonia (see sec. 16.6 of [100]) show some behavioral threshold: immediately faced with life-threatening danger, most people struggle for their own survival or that of close relatives.

Herding — Herding in a broad context means “go with the flow” or “follow the crowd”. Like “panic”, the term “herding” is often used in the context of stock market crashes, i.e. causing an avalanche effect. Like “panic” the term is usually not well defined and used in an allegoric way. Therefore, it is advisable to avoid the term in a scientific context (apart from zoology, of course). Furthermore, “herding”, “stampede”, and “panic” have a strong connotation of “deindividuation”. The conjecture of an automatic deindividuation caused by large crowds [101] has been replaced by a social attachment theory (“the typical response to a variety of threats and disasters is not to flee but to seek the proximity of familiar persons and places”) [109].

Stampede — Stampede is – like herding – a term from zoology where herds of large mammals like buffalos collectively run in one direction and might overrun any obstacles. This is dangerous for human observers if they cannot get out of the way. The term “stampede” is sometimes used for crowd accidents [73], too. It is furthermore assumed to be highly correlated with panic. When arguing along those lines, a stampede might be the result of “crowd panic” or vice versa.

Shock or density waves — Shock waves are reported for rock concerts [181] and religious events [2, 58]. They might result in people standing close to each other falling down. Pressures in dense crowds of up to 4,450 N/m² have been reported.

Although empirical data on crowd disasters exist, e.g. in the form of reports from survivors or even video footage, it is almost impossible to derive quantitative results from them. Models that aim at describing such scenarios

make predictions for certain counter-intuitive phenomena that should occur. In the faster-is-slower effect [57] a higher desired velocity leads to a slower movement of a large crowd. In the freezing-by-heating effect [56] increasing the fluctuations can lead to a more ordered state. For a thorough discussion we refer to [55, 57] and references therein. However, from a statistical point of view there is no sufficient data to decide the relevance of these effects in real emergency situations, also because it is almost impossible to perform “realistic” experiments.

Sources of Empirical Data on Evacuation Processes

The evacuation of a building can either be an isolated process (due to fire restricted to this building, a bomb threat, etc.) or it can be part of the evacuation of a complete area. We will focus on the single building evacuation, here. For the evacuation of complete areas, e.g. because of flooding or hurricanes, cf. [157] *and references therein*.

For passenger ships, a distinction between High Speed Craft (HSC), Ro-Ro passenger ferries, and other passenger vessels (cruise ships) is made. High Speed Craft do not have cabins and the seating arrangement is similar to aircraft. Therefore, there is a separate guideline for HSC [119]. An performance-based evacuation analysis at an early stage of design is required for HSC and Ro-Pax. There is currently no such requirement for cruiseships. For an overview over IMO’s requirements and the historical development up to 2001 cf. [28]. In addition to the five components for the overall evacuation time listed above, there are three more specific for ships: (6) preparation time (for the life-saving appliances, i.e. lifeboats, life-rafts, davits, chutes), (7) embarkation time, and (8) launching time. Therefore, the evacuation procedure on ships is more complex than for buildings. Additionally, SAR (Search And Rescue) is an integral part of ship evacuation.

For High Speed Craft, the time limit is 17 minutes for evacuation [18], for Ro-Ro passenger ships it is 60 minutes [118], and for all other passenger ships (e.g. cruise ships) it is 60 minutes if the number of main vertical zones is less or equal than five and 80 minutes otherwise [118]. For HSC, no distinction is made between assembly and embarkation phase.

For aircraft, the approach can be compared to that of HSC. Firstly, an evacuation test is mandatory and there is a time limit of 90 seconds that has to be complied to in the test [32].

In many countries there is no strict criterion for the maximum evacuation time of buildings of buildings. The requirements are based on minimum exit widths and maximum escape path lengths.

A number of real evacuations has been investigated and reports are publicly available. Among the most recent ones are: Beverly Hills Club [12], MGM Grand Hotel, [12], retail store [4], department store [1], World Trade Center [48] and www.wtc.nist.gov, high-rise buildings [144, 174], theatre [192] for buildings, High Speed Craft “Sleipner” [138] for HSC, an overview up

to 1998 [143], exit width variation [121], double deck aircraft [74], another overview from 2002 [120] for aircraft, and for trains [44, 170].

4 Modelling

A comprehensive theory of pedestrian dynamics has to take into account three different levels of behaviour (Fig. 8). At the *strategic level*, pedestrians decide which activities they like to perform and the order of these activities. With the choices made at the strategic level, the *tactical level* concerns the short-term decisions made by the pedestrians, e.g. choosing the precise route taking into account obstacles, density of pedestrians etc. Finally, the *operational level* describes the actual walking behaviour of pedestrians, e.g. their immediate decisions necessary to avoid collisions etc. The processes at the strategic and

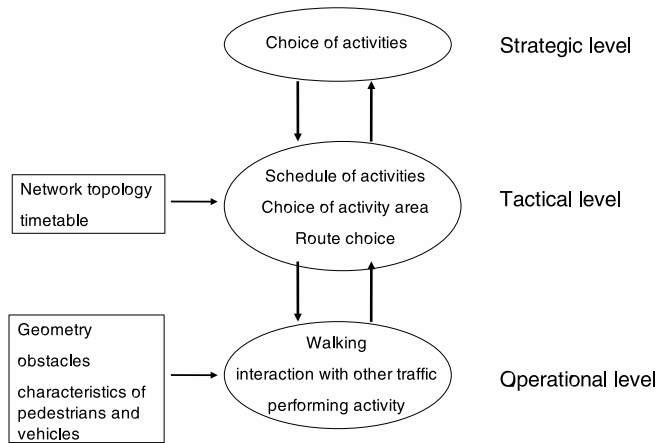


Fig. 8. The different levels of modelling pedestrian behaviour (after [20, 64]).

tactical level are usually considered to be exogenous to the pedestrian simulation. Here information from other disciplines (sociology, psychology etc.) is required. In the following we will mostly be concerned with the operational level, although some of the models that we are going to describe allow to take into account certain elements of the behaviour at the tactical level as well.

Modelling on the operational level is usually based on variations of models from physics. Indeed the motion of pedestrian crowds has certain similarities with fluids or the flow of granular materials. The goal is to find models which are as simple as possible, but at the same time can reproduce “realistic” behaviour in the sense that the empirical observations are reproduced. Therefore, based on the experience from physics, pedestrians are often modelled as simple “particles” that interact with each other.

There are several characteristics which can be used to classify the modelling approaches:

- **microscopic vs. macroscopic:** In microscopic models each individual is represented separately. Such an approach allows to introduce different types of pedestrians with individual properties as well as issues like route choice. In contrast, in macroscopic models different individuals can not be distinguished. Instead the state of the system is described by densities, usually a mass density derived from the positions of the persons and a corresponding locally averaged velocity.
- **discrete vs. continuous:** Each of the three basic variables for a description of a system of pedestrians, namely space, time and state variable (e.g. velocities), can be either discrete (i.e. an integer number) or continuous (i.e. a real number). Here all combinations are possible. In a cellular automaton approach all variables are by definition discrete whereas in hydrodynamic models all are continuous. These are the most common choices, but other combinations are used as well. Sometimes for a cellular automata approach also a continuous time variable is allowed. In computer simulation this is realized through a *random-sequential update* where at each step the particle or site to be updated (moved) is chosen randomly (from *all* particles or sites, respectively). A discrete time is usually realized through the *parallel* or *synchronous update* where all particles or sites are moved at the same time. This introduces a timescale. In so-called coupled map lattices time is discrete whereas space and state variables are continuous.
- **deterministic vs. stochastic:** The dynamics of pedestrians can either be deterministic or stochastic. In the first case the behaviour at a certain time is completely determined by the present state. In stochastic models, the behaviour is controlled by certain probabilities such that the agents can react differently in the same situation. This is one of the lessons learnt from the theory of complex systems where it has been shown for many examples that through introduction of stochasticity into rather simple systems very complex behaviour can be generated. On the other hand, the stochasticity in the models reflects our lack of knowledge of the underlying physical processes that e.g. determine the decision-making of the pedestrians. Through stochastic behavioural rules it becomes then often possible to generate a rather realistic representation of complex systems like pedestrian crowds. This “intrinsic” stochasticity should be distinguished from “noise”. Sometimes external noise terms are added to the *macroscopic* observables, like

the position or velocity. Often the main effect of these terms is to avoid certain special configurations which are considered to be unrealistic, like completely blocked states. Otherwise the behaviour is very similar to the deterministic case. For true stochasticity, on the other hand, the deterministic limit usually has very different properties from the generic case.

- **rule-based vs. force-based:** Interactions between the agents can be implemented in at least two different ways: In a rule-based approach agents make “decisions” based on their current situation and that in their neighbourhood as well as their goals etc. It focusses on the *intrinsic properties* of the agents and thus the rules are often justified from psychology. In force-based models, agents “feel” a force exerted by others and the infrastructure. They therefore emphasize the *extrinsic properties* and their relevance for the motion of the agents. It is an physical approach based on the observation that the presence of others leads to deviations from a straight motion. In analogy to Newtonian mechanics a force is made responsible for these accelerations.

Cellular automata are typically rule-based models, whereas e.g. the social-force model belongs to the force-based approaches. However, sometimes a clear distinction can not be made and many models combine aspects of both approaches.

- **high vs. low fidelity:** *Fidelity* here refers to the apparent realism of the modelling approach. High fidelity models try to capture the complexity of decision making, actions etc. that constitute pedestrian motion in a realistic way. In contrast, in the simplest models pedestrians are represented by particles without any intelligence. Usually the behaviour of these particles is determined by “forces”. This approach can be extended e.g. by allowing different “internal” states of the particles so that they react differently to the same force depending on the internal state. This can be interpreted as some kind of “intelligence” and leads to more complex approaches, like multi-agent models. Roughly speaking, the number of parameters in a model is a good measure for fidelity in the sense introduced here, but note that higher fidelity does not necessarily mean that empirical observations are reproduced better!

It should be mentioned that a clear classification according to the characteristics outlined here is not always possible. In the following we will describe some model classes in more detail.

4.1 Fluid-dynamic and gaskinetic models

Pedestrian dynamics has some obvious similarities with fluids. E.g the motion around obstacles appears to follow “streamlines”. Motion at intermediate densities is restricted (short-ranged correlations). Therefore it is not surprising that, very much like for vehicular dynamics, the earliest models of pedestrian dynamics took inspiration from hydrodynamics or gas-kinetic

theory [51, 62, 69, 70]. Typically these macroscopic models are deterministic, force-based and of low fidelity.

Henderson [61, 62] has tried to establish an analogy of large crowds with a classical gas. From measurements of motion in different crowds in a low density (“gaseous”) phase he found a good agreement of the velocity distribution functions with Maxwell-Boltzmann distribution [61].

Motivated by this observation, he has later developed a fluid-dynamic theory of pedestrian flow [62]. Describing the interactions between the pedestrians as a collision process where the particles exchange momenta and energy, a homogeneous crowd can be described by the well-known kinetic theory of gases. However, the interpretation of the quantities is not entirely clear, e.g. what the analogues of pressure and temperature are in the context of pedestrian motion. Temperature could be identified with the velocity variance, which is related to the distribution of desired velocities, whereas the pressure expresses the desire to move against a force in a certain direction.

The applicability of classical hydrodynamical models is based on several conservation laws. The conservation of mass, corresponding to conservation of the total number of pedestrians, is expressed through a continuity equation of the form

$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} + \nabla \cdot \mathbf{J}(\mathbf{r}, t) = 0, \quad (6)$$

which connects the local density $\rho(\mathbf{r}, t)$ with the current $\mathbf{J}(\mathbf{r}, t)$. This equation can be generalized to include source and sink terms. However, the assumption of conservation of energy and momentum is not true for interactions between pedestrians which in general do not even satisfy Newton’s Third Law (“actio = reactio”). In [51] several other differences to normal fluids were pointed out, e.g. the anisotropy of interactions or the fact that pedestrians usually have an individual preferred direction of motion.

In [51] a better founded fluid-dynamical description was derived on the basis of a gaskinetic model which describes the system in terms of a density function $f(\mathbf{r}, \mathbf{v}, t)$. The dynamics of this function is determined by Boltzmann’s transport equation that describes its change for a given state as difference of inflow and outflow due to binary collisions.

An important new aspect in pedestrian dynamics is the existence of desired directions of motion which allows to distinguish different groups μ of particles. The corresponding densities f_μ change in time due to four different effects:

1. A relaxation term with characteristic time τ describes tendency of pedestrians to approach their intended velocities.
2. The interaction between pedestrians is modeled by a Stosszahlansatz as in the Boltzmann equation. Here pair interactions between types μ and ν occur with a total rate that is proportional to the densities f_μ and f_ν .
3. Pedestrians are allowed to change from type μ to ν which e.g. accounts for turning left or right at a crossing.
4. Additional gain and loss terms allow to model entrances and exits where pedestrian can enter or leave the system.

The resulting fluid-dynamic equations derived from this gaskinetic approach are similar to that of ordinary fluids. However, due to the different types of pedestrians, corresponding to individuals who have approximately the same desired velocity, one actually obtains a set of coupled equations describing several interacting fluids. These equations contain additional characteristic terms describing the approach to the intended velocity and the change of fluid-type due to interactions in avoidance manoevers.

Equilibrium is approached through the tendency to walk with the intended velocity, not through interactions as in ordinary fluids. Momentum and energy are not conserved in pedestrian motion, but the relaxation towards the intended velocity describes a tendency to restore these quantities.

Unsurprisingly for a macroscopic approach, the gas-kinetic models have problems at low densities. For a discussion, see e.g. [51].

Handcalculation method

For practical applications effective engineering tools have been developed from the hydrodynamical description. In engineering these are often called *handcalculation methods*. One could also classify some of them as queing models since the central idea is to describe pedestrian dynamics as flow on a network with links of limited capacities. These methods allow to calculate evacuation times in a relatively simple way that does not require any simulations. Parameters entering in the calculations can be adapted to the situation that is studied. Often they are based on empirical results, e.g. evacuation trials. Details about this kind of models can be found in Sec. 5.1.

4.2 Social-Force Models

The social-force model [59] is a deterministic continuum model in which the interactions between pedestrians are implemented by using the concept of a *social force* or *social field* [103]. It is based on the idea that changes in behaviour can be understood in terms of fields or forces. Applied to pedestrian dynamics the social force $\mathbf{F}_j^{(\text{soc})}$ represents the influence of the environment (other pedestrians, infrastructure) and changes the velocity \mathbf{v}_j of pedestrian j . Thus it is responsible for acceleration which justifies the interpretation as a force. The basic equation of motion for a pedestrian of mass m_j is then of the general form

$$\frac{d\mathbf{v}_j}{dt} = \mathbf{f}_j^{(\text{pers})} + \mathbf{f}_j^{(\text{soc})} + \mathbf{f}_j^{(\text{phys})} \quad (7)$$

where $\mathbf{f}_j^{(\text{soc})} = \frac{1}{m_j} \mathbf{F}_j^{(\text{soc})} = \sum_{l \neq j} \mathbf{f}_{jl}^{(\text{soc})}$ is the total (specific) force due to the other pedestrians. $\mathbf{f}_j^{(\text{pers})}$ denotes a “personal” force which makes the pedestrians attempt to move with their own preferred velocity $\mathbf{v}_j^{(0)}$ and thus acts as a driving term. It is given given by

$$\mathbf{f}_j^{(\text{pers})} = \frac{\mathbf{v}_j^{(0)} - \mathbf{v}_j}{\tau_j} \quad (8)$$

where τ_j reaction or acceleration time. In high density situations also physical forces $\mathbf{f}_{jl}^{(\text{phys})}$ become important, e.g. friction and compression when pedestrians make contact.

The most important contribution to the social force $\mathbf{f}_j^{(\text{soc})}$ comes from the territorial effect, i.e. the private sphere. Pedestrians feel uncomfortable if they get too close to others, which effectively leads to a repulsive force between them. Similar effects are observed for the environment, e.g. people prefer not to walk too close to walls.

Since social forces are difficult to determine empirically, some assumptions have to be made. Usually an exponential form is assumed. Describing the pedestrians as disks of radius R_j and position (of the center of mass) \mathbf{r}_j , the typical structure of the force between the pedestrians is described by [57]

$$\mathbf{f}_{jl}^{(\text{soc})} = A_j \exp \left[\frac{R_{jl} - \Delta r_{jl}}{\xi_j} \right] \mathbf{n}_{jl} \quad (9)$$

with $R_{jl} = R_j + R_l$, the sum of the disk radii, $\Delta r_{jl} = |\mathbf{r}_j - \mathbf{r}_l|$, the distance between the centers of mass, $\mathbf{n}_{jl} = \frac{\mathbf{r}_j - \mathbf{r}_l}{\Delta r_{jl}}$, the normalized vector pointing from pedestrian l to j . A_j can be interpreted as strength, ξ_j as the range of the interactions.

The appeal of the social-force model is given mainly by the analogy to Newtonian dynamics. For the solution of the equations of motion of Newtonian many-particle systems the well-founded molecular dynamics technique exists. However, in most studies so far the distinctions between pedestrian and Newtonian dynamics are not discussed in detail. A straightforward implementation of the equations of motion neglecting these distinctions can lead to unrealistic movement of single pedestrians. For example negative velocities in the main moving direction can not be excluded in general even if asymmetric interactions (violating Newton's Third Law) between the pedestrians are chosen. Another effect is the occurrence of velocities higher than the preferred velocity $v_j^{(0)}$ due to the forces on pedestrians in the moving direction. To prevent this effect additional restrictions for the degrees of freedom have to be introduced, see for example [59], or the superposition of forces has to be discarded [177]. A general discussion of the limited analogy between Newton dynamics and the social-force model as well as the consequences for model implementations is still missing.

Apart from the ad hoc introduction of interactions the structure of the social-force model can also be derived from an extremal principle [63, 67]. It follows under the assumption that pedestrian behaviour is determined by the desire to minimize a certain cost function which takes into account not only kinematic aspects and walking comfort, but also deviations from a planned route.

4.3 Cellular Automata

Cellular automata (CA) are rule-based dynamical models that are discrete in space, time and state variable which in the case of traffic usually corresponds to the velocity. The discreteness in time means that the positions of the agents are updated in well defined steps. In computer simulations this is realized through a *parallel* or *synchronous* update where all pedestrians move at the same time. The timestep corresponds to a natural timescale Δt which could e.g. be identified with some reaction time. This can be used for the calibration of the model which is essential for making quantitative predictions. A natural space discretization can be derived from the maximal densities observed in dense crowds which gives the minimal space requirement of one person. Usually each cell in the CA can only be occupied by one particle (exclusion principle) so that this space requirement can be identified with the cell size. In this way, a maximal density of 6.25 P/m^2 [193] leads to a cell size of $40 \times 40 \text{ cm}^2$. Sometimes finer discretizations are more appropriate (see Sec. 4.5). In this case pedestrians correspond to extended particles that occupy more than one cell (e.g. four cells). The exclusion principle and the modelling of humans as non-compressible particles mimicks short-range repulsive interactions, i.e. the “private-sphere”.

The dynamics is usually defined by rules which specify the transition probabilities for the motion to one of the neighbouring cells (Fig. 9). The models differ in the specification of these probabilities as well in that of the “neighbourhood”. For deterministic models all except of one probability are zero.

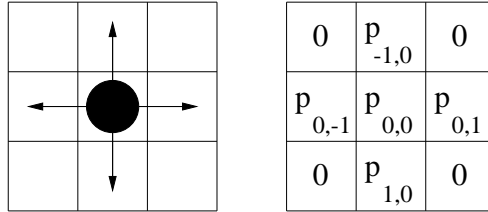


Fig. 9. A particle, its possible directions of motion and the corresponding transition probabilities p_{ij} for the case of a von Neumann neighbourhood.

The first cellular automata (CA) models [7, 42, 90, 127] for pedestrian dynamics can be considered two-dimensional variants of the asymmetric simple exclusion process (ASEP) (for reviews, see [9, 24, 173]) or models for city or highway traffic [6, 16, 133] based on it. Most of these models represent pedestrians by particles without any internal degrees of freedom. They can move to one of the neighbouring cells based on certain transition probabilities which are determined by three factors: (1) the desired direction of motion, e.g. to

find the shortest connection, (2) interactions with other pedestrians, and (3) interactions with the infrastructure (walls, doors, etc.).

Fukui-Ishibashi model

One of the first CA models for pedestrian dynamics has been proposed by Fukui and Ishibashi [41, 42] and is based on a two-dimensional variant of the ASEP. They have studied bidirectional motion in a long corridor where particles moving in opposite directions are updated alternately. Particles move deterministically in their desired direction, only if the desired cell is occupied by an oppositely moving particle they make a random sidestep.

Various extensions and variations of the model have been proposed, e.g. an asymmetric variant [127] where walkers prefer lane changes to the right, different update types [194], simultaneous (exchange) motion of pedestrians standing “face-to-face” [72], or the possibility of backstepping [107]. The influence of the shape of the particles has been investigated in [132]. Also other geometries [129, 182] and extensions to full 2-dimensional motion have been studied in various modifications [106, 107, 128]

Blue-Adler model

The model of Blue and Adler [7, 8] is based on a variant of the Nagel-Schreckenberg model [133] of highway traffic. Pedestrian motion is considered in analogy to a multi-lane highway. The structure of the rules is similar to the basic two-lane rules suggested in [159]. The update is performed in four steps which are applied to all pedestrians in parallel. In the first step each pedestrian chooses a preferred lane. In the second step the lane changes are performed. In the third step the velocities are determined based on the available gap in the new lanes. Finally, in the fourth step the pedestrians move forward according to the velocities determined in the previous step.

In counterflow situations head-on-conflicts occur. These are resolved stochastically and with some probability opposing pedestrians are allowed to exchange positions within one timestep. Note that the motion of a single pedestrian (not interacting with others) is deterministic otherwise.

Different from the Fukui-Ishibashi model motion is not restricted to nearest-neighbour sites. Instead pedestrians can have different velocities v_{\max} which correspond to the maximal number of cells they are allowed to move forward. In contrast to vehicular traffic, acceleration to v_{\max} can be assumed to be instantaneous in pedestrian motion.

In order to study the effects of inhomogeneities, the pedestrians are assigned different maximal velocities v_{\max} . Fast walkers have $v_{\max} = 4$, standard walkers $v_{\max} = 3$ and slow walkers $v_{\max} = 2$. The cell size is assumed to be 50 cm \times 50 cm. The best agreement with empirical observations has been achieved with 5% slow and 5% fast walkers [8]. Furthermore the fundamental diagram in more complex situations, like bi- or four-directional flows have been investigated.

Gipps-Marksjös model

A more sophisticated discrete model has been suggested by Gipps and Marksjös [46] already in 1985. One motivation for developing a discrete model was the limited computer power at that time. Therefore a discrete model, which reproduces the properties of pedestrian motion realistically, was in many respects a real improvement over the existing continuum approaches.

Interactions between pedestrians are assumed to be repulsive anticipating the idea of social forces (see Sec. 4.2). The pedestrians move on a grid of rectangular cells of size 0.5×0.5 m. To each cell a score is assigned based on its proximity to other pedestrians. This score represents the repulsive interactions and the actual motion is then determined by the competition between these repulsion and the gain of approaching the destination. Applying this procedure to all pedestrians, to each cell a potential value is assigned which is the sum of the individual contributions. The pedestrian then selects the cell of its nine neighbours (Moore neighbourhood) which leads to the maximum benefit. This benefit is defined as the difference between the gain of moving closer to the destination and the cost of moving closer to other pedestrians as represented by the potential. This requires a suitable chosen gain function P .

The updating is done sequentially to avoid conflicts of several pedestrians trying to move to the same position. In order to model different velocities, faster pedestrians are updated more frequently. Note that the model dynamics is deterministic.

Floor field CA

The floor field CA [13, 14, 86, 167] can also be considered as an extension of the ASEP. However, the transition probabilities to neighbouring cells are no longer fixed but vary dynamically. This is motivated by the process of chemotaxis (see [5] for a review) used by some insects (e.g. ants) for communication. They create a chemical trace to guide other individuals to food sources. In this way a complex trail system is formed that has many similarities with human transport networks.

In the approach of [14] the pedestrians also create a trace. In contrast to chemotaxis, however, this trace is only virtual although one could assume that it corresponds to some abstract representation of the path in the mind of the pedestrians. Although this is mainly a technical trick which reduces interactions to local ones that allow efficient simulations in arbitrary geometries, one could also think of the trail as representation of the paths in the mind of a pedestrian. The locality becomes important in complex geometries as no algorithm is required to check whether the interaction between particles is screened by walls etc. The number of interaction terms always grows linearly with the number of particles.

The translation into local interactions is achieved by the introduction of so-called *floor fields*. The transition probabilities for all pedestrians depend on

the strength of the floor fields in their neighbourhood in such a way that transitions in the direction of larger fields are preferred. The *dynamic floor field* D_{ij} corresponds to a virtual trace which is created by the motion of the pedestrians and in turn influences the motion of other individuals. Furthermore it has its own dynamics, namely through diffusion and decay, which leads to a dilution and finally the vanishing of the trace after some time. The *static floor field* S_{ij} does not change with time since it only takes into account the effects of the surroundings. Therefore it exists even without any pedestrians present. It allows to model e.g. preferred areas, walls and other obstacles. Fig. 10 shows the static floor field used for the simulation of evacuations from a room with a single door. Its strength decreases with increasing distance from the door. Since the pedestrian prefer motion into the direction of larger fields, this is already sufficient to find the door.

Coupling constants control the relative influence of both fields. For a strong coupling to the static field pedestrians will choose the shortest path to the exit. This corresponds to a 'normal' situation. A strong coupling to the dynamic field implies a strong herding behaviour where pedestrians try to follow the lead of others. This often happens in emergency situations.

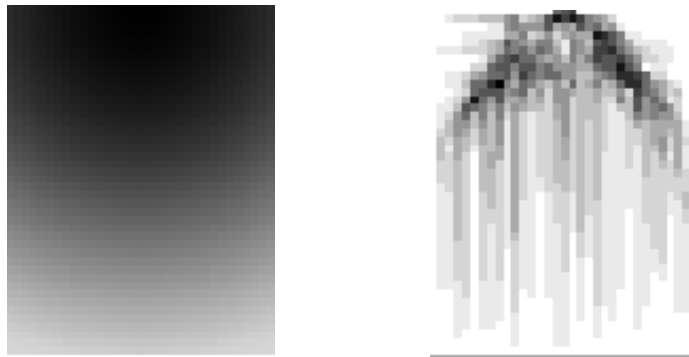


Fig. 10. Left: Static floor field for the simulation of an evacuation from a large room with a single door. The door is located in the middle of the upper boundary and the field strength is increasing with increasing intensity. Right: Snapshot of the dynamical floor field created by people leaving the room.

The model uses a fully parallel update. Therefore conflicts can occur where different particles choose the same destination cell. This is relevant for high density situations and happens in all models with parallel update if motion in different directions is allowed. Conflicts have been considered a technical problem for a long time and usually the dynamics has been modified in order to avoid them. The simplest method is to update pedestrians sequentially instead of using a fully parallel dynamics. However, this leads to other problems, e.g.

the identification of the relevant timescale. Therefore it has been suggested in [84] to take these conflicts seriously as an important part of the dynamics.

For the floor field model it has been shown in [85] that the behaviour becomes more realistic if not all conflicts are resolved in the sense that one of pedestrian is allowed to move whereas the others stay at their positions. Instead with probability $\mu \in [0, 1]$, which is called friction parameter, the movement of *all* involved pedestrians is denied [85] (see Fig. 11). This allows

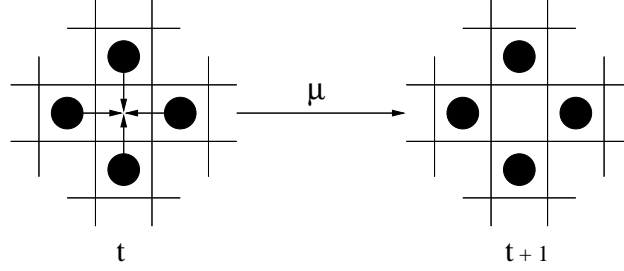


Fig. 11. Refused movement due to the friction parameter μ (for $m = 4$).

to describe clogging effects between the pedestrians in a much more detailed way [85]. μ works as some kind of local pressure between the pedestrians. If μ is high, the pedestrians handicap each other trying to reach their desired target sites. This local effect can have enormous influence on macroscopic quantities like flow and evacuation time [85]. Note that the kind of friction introduced here only influences interacting particles, not the average velocity of a freely moving pedestrian.

Surprisingly the qualitative behaviour of the floor field model and the social-force model is very similar despite the fact that the interactions are very different. In the floor field model interactions are attractive whereas they are repulsive in the social-force model. However, in the latter interactions are between particle densities. In contrast in the floor field model the particle density interacts with the velocity density.

4.4 Other Approaches

Lattice-gas models

In 1986, Frisch, Hasslacher, and Pomeau [35] have shown that one does not have to take into account the detailed molecular motion within fluids in order to obtain a realistic picture of (2d) fluid dynamics. They proposed a lattice gas model [164, 165] on a triangular lattice with hexagonal symmetry which is similar in spirit to CA models, but the exclusion principle is relaxed: Particles with different velocities are allowed to occupy the same site. Note that the allowed velocities differ only in the direction, not the absolute value. The dynamics is based on a succession of collision and propagation that can be chosen

in such a way that the coarse-grained averages of this microscopic dynamics is asymptotically equivalent to the Navier-Stokes equations of incompressible fluids.

In [108] a kind of mesoscopic approach inspired by these lattice gas models has been suggested as a model for pedestrian dynamics. In analogy with the description of transport phenomena in fluids (e.g. the Boltzmann equation) the dynamics is based on a succession of collision and propagation. Pedestrians

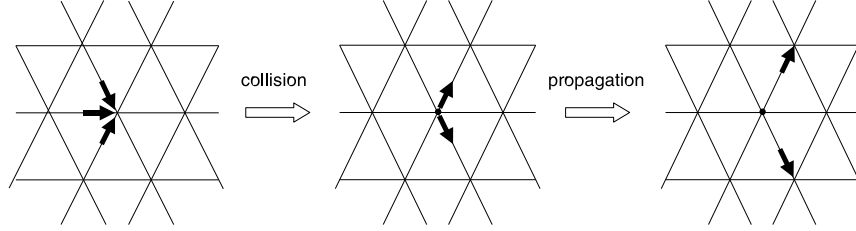


Fig. 12. The dynamics of lattice gas models proceeds in two steps. Pedestrians coming from neighbouring sites interact in the collision step where velocities are redistributed. In the propagation step the pedestrians move to neighbour sites in the directions determined in the collision step.

are modelled as particles moving on a triangular lattice which have a preferred direction of motion \mathbf{c}_F . However, the particles do not follow strictly this direction but have also a tendency to move with the flow. Furthermore at high densities the crowd motion is influenced by a kind of friction which slows down the pedestrians. This is achieved by reducing the number of individuals allowed to move to neighbouring sites.

As in a lattice gas model [165], the dynamics now consists of two steps. In the *propagation step* each pedestrian moves to the neighbour site in the direction of its velocity vector. In the *collision step* the particles interact and new velocities (directions) are determined. In contrast to physical systems, momentum etc. does not need to be conserved during the collision step. These considerations lead to a collision step that takes into account the favorite direction \mathbf{c}_F , the local density (the number of pedestrians at the collision site), and a quantity called mobility at all neighbour sites which is a normalized measure of the local flow after the collision.

Optimal-velocity model

The optimal velocity (OV) model originally introduced for the description of highway traffic can be generalized to higher dimensions [134] which allows its application to pedestrian dynamics.

In the two-dimensional extension of the OV model the equation of motion for particle i is given by

$$\frac{d^2}{dt^2}\mathbf{x}_i(t) = a \left\{ \mathbf{V}_0 + \sum_j \mathbf{V}(\mathbf{x}_j(t) - \mathbf{x}_i(t)) - \frac{d}{dt}\mathbf{x}_i(t) \right\}, \quad (10)$$

where $\mathbf{x}_i = (x_i, y_i)$ is the position of particle i . It can be considered as a special case of the general social-force model (7) without physical forces. The optimal-velocity function

$$\mathbf{V}(\mathbf{x}_j - \mathbf{x}_i) = f(r_{ij})(1 + \cos \varphi) \mathbf{n}_{ij}, \quad (11)$$

$$f(r_{ij}) = \alpha \{ \tanh \beta(r_{ij} - b) + c \}, \quad (12)$$

where $r_{ij} = |\mathbf{x}_j - \mathbf{x}_i|$, $\cos \varphi = (x_j - x_i)/r_{ij}$ and $\mathbf{n}_{ij} = (\mathbf{x}_j - \mathbf{x}_i)/r_{ij}$ is determined by interactions with other pedestrians. \mathbf{V}_0 is a constant vector that represents a ‘desired velocity’ at which an isolated pedestrian would move. The strength of the interaction depends on the distance r_{ij} between the i th and j th particles, and on the angle φ between the directions of $\mathbf{x}_j - \mathbf{x}_i$ and the current velocity $\frac{d}{dt}\mathbf{x}_i$. Due to the term $(1 + \cos \varphi)$, a particle reacts more sensitively to particles in front than those behind.

Now two cases can be distinguished, namely repulsive and attractive interactions. The former is relevant for pedestrian dynamics whereas the latter is more suitable for biological motion. Therefore for pedestrian motion one chooses $c = 1$ which implies $f < 0$.

A detailed analysis [134] shows that the model exhibits a rich phase diagram including the formation of various patterns.

Other models

We briefly mention a few other model approaches that have been suggested. In [10] a discretized version of the social-force model has been introduced and shown to reproduce qualitatively the observed collective phenomena.

In [141] a magnetic force model has been proposed where pedestrians and their goals are treated as magnetic poles of opposite sign.

Another class of models is based on ideas from queuing theory. In principle, some handcalculation methods can be considered as a macroscopic queuing model. Typically rooms are represented as nodes in the queuing network and links correspond to doors. In microscopic approaches in the movement process each agent chooses a new node, e.g. according to some probability [105].

4.5 Theoretical Results

As emphasized in Sec. 3.2, the collective effects observed in the motion of pedestrian crowds are a direct consequence of the microscopic dynamics. These effects are reproduced quite well by some models, e.g. the social-force and floor-field model, at least on a qualitative level. As mentioned before, the qualitative behaviour of the two models is rather similar despite the very different implementation of the interactions. This indicates a certain robustness of the collective phenomena observed.

As an example we discuss the formation of lanes in counterflow formation. Empirically one observes a strong tendency to follow immediately in the “wake” of another person heading into the same direction. Such lane formation was reproduced in the social-force model [56, 59] as well as in the floor-field model [14, 76] (see Fig. 13). While the formation of lanes in general is essential to avoid deadlocks and thus keep the chance to reproduce realistic fluxes, the number of direction changes per meter cross section is a parameter which in reality crucially depends on the situation [76]: The longer a counterflow situation is assumed to persist, the less lanes per meter cross section can be found. The correct reproduction of counterflow is an issue for an accommodating animation, but more or less unimportant for the macroscopic observables. This is probably the main reason why there seems to have been not much effort put into the attempt to reproduce different “kinds” of lane formation in a controlled, situation-dependent manner.

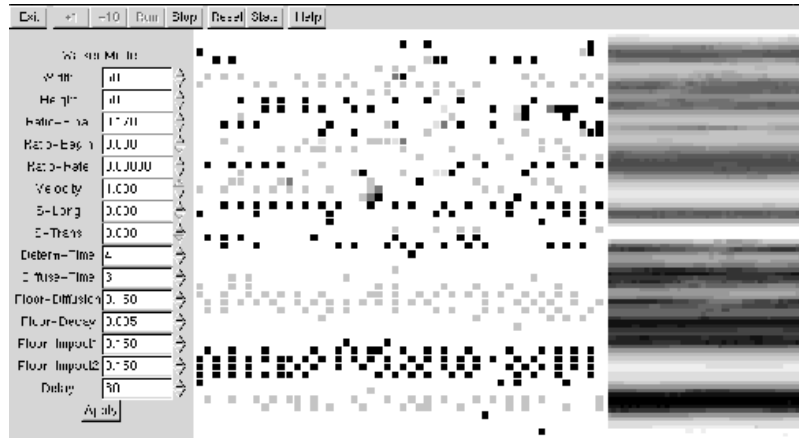


Fig. 13. Lane formation in the floor-field model. The central window is the corridor and the light and dark squares are right- and left-moving pedestrians, respectively. In the bottom part well-separated lanes can be observed whereas in the top part the motion is still disordered. The right part of the figure shows the floor fields for the right-movers (upper half) and left-movers (lower half).

On the quantitative side, the fundamental diagram is the first and most serious test for any model. Since most quantitative results rely on the fundamental diagram it can be considered the most important characteristics of pedestrian dynamics. It is not only relevant for movement in a corridor or through a bottleneck, but also as an important determinant of evacuation times. However, as emphasized earlier, there is currently no consensus on the empirical form of the fundamental diagram. Therefore a calibration of the model parameters is currently not sensible.

Most cellular automata models are based on the asymmetric simple exclusion process. This strictly one-dimensional stochastic process has a fundamental diagram which is symmetric around density $\rho = 1/2$. Lane changes in two-dimensional extensions lead only to a small shift towards smaller densities. Despite the discrepancies in the empirical results, an almost symmetric fundamental diagram can be excluded.

Based on the experience with modelling of highway traffic [16, 133] therefore models with higher speeds have been introduced which naturally lead to an asymmetric fundamental diagram. Typically this is implemented by allowing the agents to move more than one cell per update step [82, 83, 87, 95, 196, 197]. These model variants have been shown to be flexible enough to reproduce e.g. Weidmann’s fundamental diagram for the flow in a corridor [193] with high precision. Usually in the simulations a homogeneous population is assumed. However in reality different pedestrians have different properties like walking speed, motivation etc. This is easily taken into account in every microscopic model. There are many parameters that could potentially have an influence on the fundamental diagram. However, the current empirical situation does not allow to decide this question.

Another problem occurring in CA models has its origin in the discreteness of space. Through the choice of the lattice discretization space is no longer isotropic. Motion in directions not parallel to the main axis of the lattice are difficult to realize and can only be approximated by a sequence of steps parallel to the main directions.

Higher velocities require also the extension of the neighbourhood of a particle which is no longer identical to the cells adjacent to the current position. A natural definition of “neighbourhood” corresponds to those cells that could be reached within one timestep. In this way the introduction of higher velocities also reduces the problem of space isotropy as the neighbourhoods become more isotropic for larger velocities.

Other solutions to this problem have been proposed. One way is to count the number of diagonal steps and let the agent suspend from moving following certain rules which depend on the number of diagonal steps [172]. A similar idea is to sum up the real distance that an agent has moved during one round: A diagonal step counts $\sqrt{2}$ and a horizontal or vertical one 1. An agent has to finish its round as soon as this sum is bigger than its speed [87]. A third possibility - which works for arbitrary speeds - is to assign selection probabilities to each of the four lattice positions which are adjacent to the exact final position [196, 197]. Naturally these probabilities are proportional to the square area between the exact final position and the lattice point, as in this case the probabilities are normalized by construction if one has a square lattice with points on all integer number combinations. However, one also could think of other methods to calculate the probability.

For the social-force model, the specification of the repulsive interaction (with and without hard core, exponential or reciprocal with distance) as well as the parameter sets for the forces changes in different publications

[56, 57, 59, 114]. In [55] the authors state that “most observed self-organization phenomena are quite insensitive to the specification of interaction forces”. However, at least for the fundamental diagram, a relation connected with all phenomena in pedestrian dynamics, this statement is questionable. As remarked in [54] the reproduction of the fundamental diagram “requires a less simple specification of the repulsive interaction forces”. Indeed in [177] it was shown that the choice of hard-core forces or repulsive soft interactions as well as the particular parameter set can strongly influence the resulting fundamental diagram regarding qualitative as well as quantitative effects.

Also a more realistic behaviour at higher densities requires a modification of the basic model. Here the use of density-dependent desired velocities leads to a reduction of the otherwise unrealistically large number of collisions [10].

The particular specification of forces and the previously mentioned problem with Newton’s Third law can lead in principle to some unwanted effects, like momentary velocities larger than the preferred velocity [59] or the penetration of pedestrians into each other or into walls [98]. It is possible that these effects can be suppressed for certain parameter sets by contact or friction forces, but the general appearance is not excluded. Only in the first publication [59] restrictions for the velocity are explicitly formulated to prevent velocities larger than the intended speed and other authors tried to improve the model by introducing more parameters [98]. But additional parameter and artificial restrictions of variables diminish the simplicity and thus the attractiveness of the model. A general discussion how to deal with these problems of the social-force model and a verification that the observed phenomena are not limited to a certain specification of the interaction and a special parameter set is up to now still missing.

While the realistic reproduction within the empirical range of these macroscopic observables, especially the fundamental diagram, is absolutely essential to guarantee safety standards in evacuation simulations, and while a user should always be distrustful of models where no fundamental diagram has ever been published, it is by no means sufficient to exclusively check for the realism of macroscopic observables. On the microscopic level there is a large amount of phenomena which need to be reproduced realistically, be it just to make a simulation animation look realistically or be it for the reason that microscopic effects can often easily influence macroscopic observables.

If one compares simulations of bottleneck flows with real events, one observes that in simulations the form of the queue in front of bottlenecks is often a half-circle, while in reality it is drop- or wedge-shaped. In most cases this discrepancy probably does not have an influence on the simulated evacuation time, but it is interesting to note, where it originates from. Most simulation models implicitly or explicitly use some kind of utility maximization to steer the pedestrians – with the utility being foremost inversely proportional to the distance from the nearest exit. This obviously leads to half-circle-shaped queues in front of bottlenecks. So wherever one observes queues different than half-circles, people have exchanged their normal “utility function based on the

distance” with something else. One such alternative utility function could be that people are just curious what is inside or behind the bottleneck, so they need to seek a position where they can look into it. A more probable explanation would be that in any case it is the time distance not the spatial distance which is sought to be minimized. As anyone knows about the inescapable loss in time a bottleneck means for the whole waiting group, the precise waiting spot is not that important. However, in societies with a strong feeling for equality, people strongly would wish to equally distribute the waiting time and keep a first-in-first-out principle, which can best be accomplished and controlled when the queue is more or less one-dimensional, respectively just as wide as the bottleneck itself.

Finally it should be mentioned that theoretical investigations based on simulations of models for pedestrian dynamics have lead to the prediction of some surprising and counter-intuitive collective phenomena, like the reduction of evacuation times through additional columns near exits (see Sec. 3.5) or the faster-is-slower [57] and freezing-by-heating effect [56]. However, so far the empirical evidence for the relevance or even occurrence of these effects in real situations is rather scarce.

5 Applications

In the following section we discuss more practical aspects of based on the modelling concepts presented in Sec. 4. Tools of different sophistication have been developed that are nowadays routinely used in safety analysis. The latter becomes more and more relevant since many public facilities have to fulfill certain legal standards. As an example we mention aircrafts which have to be evacuated within 90 seconds. The simulations etc. are already used in the planning stages because changes of the design at a later stage are difficult and expensive.

For this kind of safety analysis tools of different sophistication have been developed. Some of them mainly are able to predict just evacuation times whereas others are based on microscopic simulations which allow also to study various external influences (fire, smoke, ...) in much detail.

5.1 Calculation of Evacuation Times

The basic idea of handcalculation methods has already briefly been described at the end of Sec. 4.1. Here we want to discuss its practical aspects in more detail.

The approach has been developed since the middle of the 1950s [186]. The basic idea of these methods is the assumption that people can be calculated or behave like fluids. Knowledge of the flow (see Equ. 1) and the technical data of the facility are then sufficient to evaluate evacuation times etc.

Handcalculation methods can be divided in two major approaches: methods with “dynamic” flow [36, 43, 78–80, 136, 151, 152, 163, 193] and methods with “fixed” flow [110, 123–126, 137, 145, 174, 186]. As methods with “dynamic” flow we call methods where the pedestrian flow is dependent from the density of the pedestrian stream (see Sec. 3.3) in the selected facility, thus the flow can be obtained from fundamental diagrams (see Sec. 3.4) or it is explicitly prescribed in the chosen method. This flow can change during movement through the building, e.g. by using stairs, thus the pedestrian stream has a “dynamic” flow. Methods with “fixed” flow do not use this concept of relationship between density and flow. In this methods selected facilities (e.g. stairs or doors) has a fixed flow which is independent from the density, that is usually not used in this methods. The “fixed” flow usually based upon empirical and measured data of flow, which are specified for a special type of building, like high-rise buildings or railway stations, for example. Because of much simplifications in these “fixed” flow methods a calculation can always be done very fast.

Methods with “dynamic” flow allow the user to describe the condition of the pedestrian flow in every part of the selected building or environment, because they are mostly based upon the continuity equation, thus it is possible to calculate different kind of buildings. This allows the user to calculate transitions from wide to narrow, floor to door, floor to stair, etc. The disadvantage is that some these methods are very elaborate and time-intensive. But not in general a method with “dynamic” flow is complicated to calculate, thus we want to divide handcalculation methods in simple [36, 43, 110, 123–126, 136, 137, 145, 151, 163, 174, 186, 193] and complex [78–80, 152] for evacuation calculation. All of these handcalculation methods are able to predict total evacuation times for a selected building, but differences between different methods are still alive. Thus the user has to ensure that he is familiar with assumptions made by each method to ensure that a result is interpreted in a correct way [161].

5.2 Simulation of Evacuation Processes

Before we go into the details of evacuation simulation, let us briefly clarify its scope and limitations and contrast it to other methods used in evacuation analysis. When analyzing evacuation processes, three different approaches can be identified: (1) risk assessment, (2) optimization, and (3) simulation. The aim and result of risk-assessment is a list of events and their consequences (e.g. damage, financial loss, loss of life), i.e. usually an event tree with probabilities and expectation values for financial loss. Optimization aims at, roughly speaking, minimizing the evacuation time and reducing the area and duration of congestion. And finally, simulation describes a system with respect to its function and behavior by investigating a model of the system. This model is usually non-analytic, does not provide explicit equations for the calculation of, e.g. evacuation time. Of course, simulations are used for “optimization” in a more general sense, too, i.e. they can be part of an optimization. This holds

for risk assessment, too, if simulations are used to determine the outcomes of the different scenarios in the event tree.

In evacuation analysis the system is, generally speaking, a group of persons in an environment. More specifically, four components (sub-systems/sub-models) of the system *evacuation process* can be identified: (1) geometry, (2) environment, (3) population, and (4) hazards [44]. Any evacuation simulation must at least take into account (1) and (3). The behavior of the persons (which can be described on the strategic, tactical, and operational level — see Sec. 4) level is part of the population sub-model. An alternative way of describing the behavior is according to its algorithmic representation: no behavior modeling - functional analogy - implicit representation (equation) - rule based - artificial intelligence [44].

Hazards are in the context of evacuation first of all fire and smoke, which then require a toxicity sub-model, e.g. the fractional effective dose model (FED), to assess their physiological effect of toxic gases and temperature [26]. Further hazards to take into account might be earthquakes, floodings, or in the case of ships, list, heel, or roll motion. The sub-model environment comprises all other influences that affect the evacuation process, e.g. exit signs, surface texture, public address system, etc.

In summary, aims of an evacuation analysis and simulation are to provide feedback and hints for improvement at an early stage of design, information for safer and more rigorous regulations, improvement of emergency preparedness, training of staff, and accident investigation [44]. They usually do not provide direct results on the probability of a scenario or a systematic search for optimal geometries.

Calculation of Overall Evacuation Time, Identification of Congestion, and Corrective Actions

The scope of this section is to show general results that can be obtained by evacuation simulations. They are general in the sense that they can basically be obtained by any stochastic and microscopic model, i.e. apart from these two requirements, the results are not model specific. In detail, five different results of evacuation simulations can be distinguished: (1) distribution of evacuation times, (2) evacuation curve (number of persons evacuated vs. time), (3) sequence of the evacuation (e.g. snapshots/screenshots at specific times, e.g. every minute), and (4) identification of congestion, usually based on density and time. Especially the last point (4) needs some more explanation: Congestion is defined based on density. Notwithstanding the difficulties when measuring density, we suggest density as the most suitable criterion for the identification of congestion. In addition to the mere occurrence of densities exceeding a certain threshold (say 3.5 persons per square meter), the time this threshold is exceeded is another necessary condition for a sensible definition of congestion. In the case presented here, 10% of the overall evacuation time is used. Both criteria are in accordance with the IMO regulations [118].

Based on these results, evacuation time and areas of congestion, corrective actions can be taken. The most straightforward measure would be a change of geometry, i.e. shorter or wider escape paths (floors, stairs, doors). This can be directly put into the geometry sub-model, the simulation be re-run, and the result checked. Secondly, the signage and therefore the orientation capability could be improved. This is not as straightforward as geometrical changes. It does depend more heavily on the model characteristics how these changes influence the evacuation sequence.

We will not go into these details in the following two sections but rather show two typical examples for evacuation simulations and the results obtained. We will also not discuss the results in detail, since they are of an illustrative nature in the context of this article. The following examples are based on investigations that have been performed using a cellular automaton model which is described along with the simulation program in [89,111].

Simulation Example 1 - Hotel

The first example we show is a hotel with 8069 persons. In fig. 15 only the ground floor is shown. There are nine floors altogether. The upper floors influence the ground floor only via the stair landings and the exits adjacent to them. Most of the 8069 persons are initially located in the ground floor, since the theatre and conference area is located there. The upper floors are mainly covering bedrooms and some small conference areas.

The first step in our example (which might well be a useful recipe for evacuation analyses in general and is again in accordance with [118]) is to perform a statistical analysis. To this end, 500 samples are simulated. The evacuation time of a single run is the time it takes for all persons to get out. In this context, no fire or smoke are taken into account. Since there are stochastic influences in the model used, the significant overall evacuation time is taken to be the 95-percentil (cf. fig. 14). Finally, the maximum, minimum, mean, and significant values for the evacuation curve (number of persons evacuated vs. time) are shown in fig. 14, too.

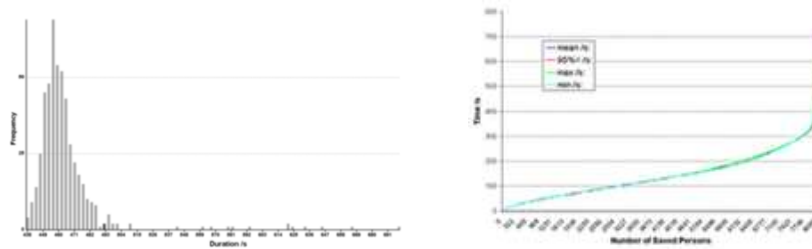


Fig. 14. Frequency distribution and evacuation curve.



Fig. 15. Initial population distribution and situation after two minutes.

The next figure (fig. 16) shows the cumulated density. The thresholds (red areas) are 3.5 persons per square meter and 10% of the overall evacuation time (in this case 49 seconds). The overall evacuation time 8:13 minutes (493 seconds). This value is obtained by taking the 95-percentile of the frequency distribution for the overall evacuation times (cf. fig. 14).

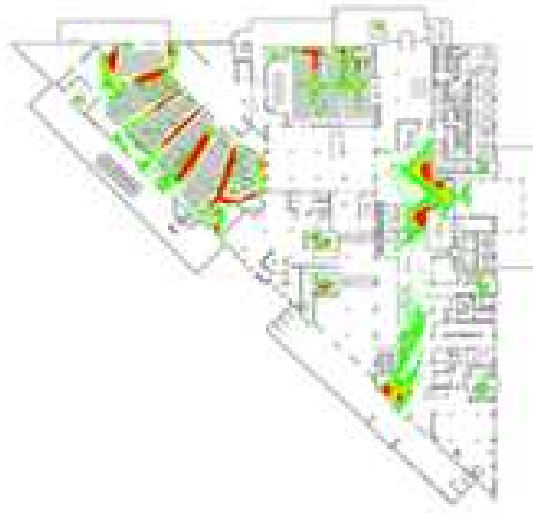


Fig. 16. Density plot, i.e. cumulated person density exceeding 3.5 persons per square meter and 10% of overall evacuation time.

Of course, a distribution of overall evacuation times (for one scenario, i.e. the same initial parameters) can only be obtained by a stochastic model. In a deterministic model only one single value is calculated for the overall evacuation time. The variance of the overall evacuation times is due to two effects in the model used here: the initial position of the persons is determined anew at the beginning of each simulation run since only the statistical properties of the overall population is set and the motion of the persons is governed by partially stochastic rules (e.g. probabilistic parameters).

Simulation Example 2 - Passenger Ship

The second example we will show is a ship. The major difference to the previous example is the distinction of (1) assembly phase and (2) embarkation and launching.

$$T = A + \frac{2}{3}(E + L) = f_{\text{safety}} \cdot (t_{\text{react}} + t_{\text{walk}}) + \frac{2}{3}(E + L) \leq 60 \text{ minutes.}$$

Embarkation and launching time (E+L) are required to be less than 30 minutes. For the sake of the evacuation analysis at an early design stage, the sum of embarkation and launching time can be assumed to be 30 minutes. Therefore, the requirement for A is 40 minutes. Alternatively, the embarkation and launching time can be determined by an evacuation trial.

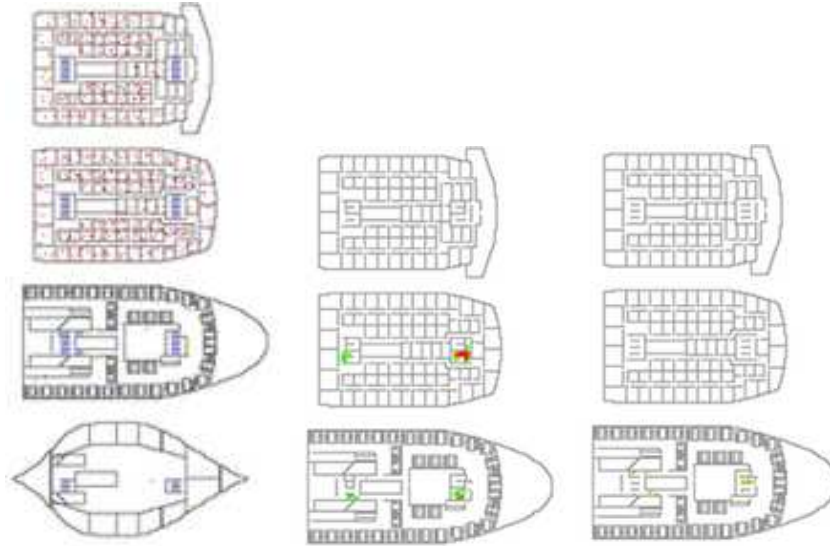


Fig. 17. Initial distribution for the night case, density plot for the day case, and density plot for the night case for the “AENEAS steamliner”.

Figure 17 shows the layout, initial population distribution (night case), density plot for the day case, and density plot for the night case. The reaction times are different for the day and the night case: 3 to 7 minutes (equally distributed) in the one and 7 to 13 minutes in the other case. The longer reaction time in the night case results in less congestion (cf. Fig. 17). Both cases have to be done in the analysis according to [118]. Additionally, a secondary night and day case are required (making up for four cases altogether): In these secondary cases the main vertical zone (MVZ) leading to the longest overall individual assembly time is identified, and then either half of the stairway capacity in this zone is assumed to be not available, or 50% of the persons initially located in this zone have to be lead via one neighbouring zone to the assembly station.

In the same way as shown for the two examples, simulations can be performed for other types of buildings and vessels. It has been applied to various passengers ships [112] to football stadiums [88] and the World Youth Day 2005 [88], the Jamarat Bridge in Makkah [88], a movie theater and schools (mainly for calibration and validation) [89] and airports [172]. Of course, many examples of application based on various models can be found in the literature. For an overview, the proceedings of the PED conference series are an excellent starting point [45, 171, 191].

5.3 Comparison of Commercial Software Tools

From a practical point of view, application of models for pedestrian dynamics and evacuation processes becomes more and more relevant in safety analysis. This has lead to the development of a number of software tools that, with different sophistication, to study many aspects without risking the health of test persons in evacuation trials.

Commercial and non-commercial software tools are based on different types of modeling [97, 188] and they became very popular since the middle of the 1990s. A first comparison of different commercial software tools can be found in [192], where they were attested to produce “reasonable results”. Further comparisons of real evacuation data with software tools or handcalculation methods can be found in [30, 68, 91, 96, 104, 160, 161, 178]. But results predicted by different commercial software tools can differ by up to 40% for the same building [96]. By calculating with different assumptions, e.g. different reaction times, use of more or less detailed stair models or calculating with a real occupant load in contrast to an uncertainty analysis, the results may be different, too [96, 104]. Contrary to these results another study [161] shows that calculations with different software tools are able to predict total evacuation times for high-rise buildings and there are no large differences as shown in [96]. In [161] it is also shown that commercial software tools are not able to predict “correct” evacuation times for selected floors of high-rise buildings with very low densities. In this case human behaviour has a very large influence on the evacuation time contrary to evacuations at with medium or high densities,

where human behaviour has an smaller influence on the evacuation time of selected areas, because congestions appear and continue larger than in low density situations, thus people are obtaining the exit where the congestion is still alive [162]. In low density situations congestions are very rare, thus people are moving narrowly with free walking velocity through the building [162].

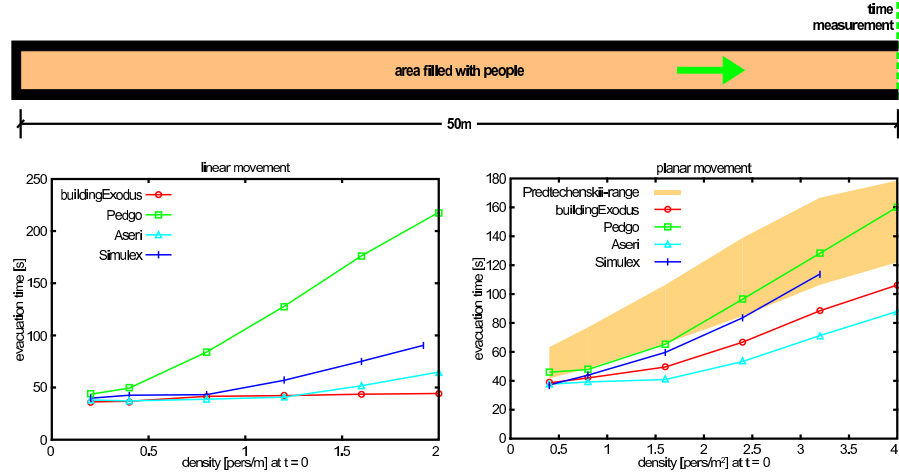


Fig. 18. Comparison of different software tools by simulating linear (left) and planar (right) movement [162]

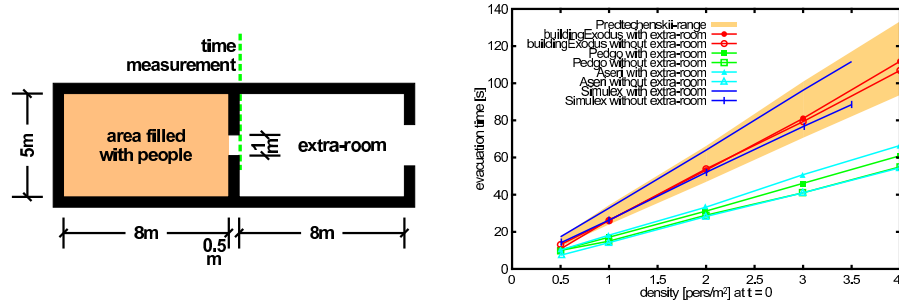


Fig. 19. Comparison of different software tools by simulating a simple room geometry [162]

But the results presented in [161] also show that commercial software tools have sometimes problems with the empirical relationship of density and walking speed (see Fig. 18). Furthermore it is very important how boundary

conditions are implemented in these tools (see Fig. 19), and the investigation of a simple scenario of a single room using different software tools shows results differing of about a factor of two (see Fig. 19) [161]. In this case all software tools predict a congestion at the exit. Furthermore it is possible that the implemented algorithm fails [161]. Thus for the user it is hard to know which algorithms are implemented in closed-source tools so that such a tool must be considered as “black box” [147]. It is also quite difficult to compare results about density and appearing congestions calculated by different software tools [162] and so it is questionable how these results should be interpreted. But, as pointed out earlier, reliable empirical data are often missing so that a validation of software tools or models is quite difficult [162].

6 Future Directions

The discussion has shown that the problem of crowd dynamics and evacuation processes is far from being well understood. One big problem is still the experimental basis. As in many human systems it is difficult to perform controlled experiments on a sufficiently large scale. This would be necessary since data from actual emergency situation is usually not available, at least in sufficient quality. Progress should be possible by using modern video and computer technology which should allow in principle to extract precise data even for the trajectories of individuals.

The full understanding of the complex dynamics of evacuation processes requires collaboration between engineering, physics, computer science, psychology etc. Engineering in cooperation with computer science will lead to an improved empirical basis. Methods from physics allow to develop simple but realistic models that capture the main aspects of the dynamics. Psychology is then needed to understand the interactions between individuals in sufficient detail to get a reliable set of ‘interaction’ parameters for the physical models.

In the end these joint efforts will hopefully lead to realistic model for evacuation processes that not only allow to study these already in the planning stages of facilities, but even allow for a dynamical real-time evacuation control in case an emergency occurs.

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